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Rational Approximations Principle for Frequency Shifts Measurement in Frequency Domain Sensors

Fabian N. Murrieta-Rico,
Andrei Tchernykh
Centro de Investigación Científica y de Educación Superior de Ensenada
Ensenada, Baja California, México
fmurriet@cicese.edu.mx
chernykh@cicese.mx

Vitalii Petranovskii,
Oscar Raymond-Herrera
Centro de Nanociencias y Nanotecnología
Universidad Nacional Autónoma de México
Ensenada, Baja California, México
vitalii@cnyn.unam.mx
raymond@cnyn.unam.mx

Oleg Sergiyenko,
Wendy Flores-Fuentes
Instituto de Ingeniería
Universidad Autónoma de Baja California
Mexicali, Baja California, México
srgnk@uabc.edu.mx

Julio C. Rodriguez-Quiñonez
Daniel Hernandez-Balbuena
Facultad de Ingeniería
Universidad Autónoma de Baja California
Mexicali, Baja California, México
julio.rodriguez81@uabc.edu.mx
dhernan@uabc.edu.mx

Juan Ivan Nieto Hipolito
Facultad de Ingeniería, Arquitectura y Diseño
Universidad Autónoma de Baja California
Ensenada, Baja California, México
jnieto@uabc.edu.mx

Vera Tyrsa
Kharkiv National Technical University of Agriculture,
Kharkiv, Ukraine
vera-tyrsa@mail.ru

Vladimir M. Kartashov
Department of Radio Electronic Systems,
Kharkiv National University of Radio Electronics,
Kharkiv, Ukraine
kartashov@kture.kharkov.ua

***Abstract*—Frequency domain sensors (FDS) are important elements in control, data acquisition and monitoring systems. Such sensors have some outstanding characteristics like output of quasi-digital signals, high sensitivity, high resolution, wide dynamic range, anti-interference capacity and good stability. A FDS converts a physical variable (measurand) into a frequency domain ouput. When the measurand changes, the output has a proportional frequency shift. In systems that use FDS, measuring the frequency shift is desirable. Accuracy of most frequency measurement techniques is limited by measurement time, and if more precision is required, longer times for measuring are needed. In this work, a novel approach using the rational approximations principle for measuring frequency shift in the output of a FDS is introduced. Also algorithms for simulating the mathematical model of frequency measurement process are proposed, and resolution of measurement is improved by analyzing the data obtained.**

***Index Terms*—frequency measurement, frequency shift, FDS**

I. Introduction

Frequency domain sensors (FDS) are used for measuring physical parameters (measurand). In FDS (also known as "frequency output sensors"), the measurand is converted into a quasi-square wave with a frequency or period [1]. After the measurand variation, a frequency shift occurs in the sensor's output. Frequency meters are used for measuring the FDS output value.

Quartz crystal microbalances and surface acoustic wave sensors are examples of FDS. When a frequency domain sensor experiments a frequency shift, the variations on its output are several orders of magnitude below the original output value. We'll briefly review some examples of frequency shifts in FDS reported on the literature. In [2], a silver-coated QCM resonator operating to 9 MHz experimented a frequency shift around 80 Hz, and frequency shifts were recorded by an Agilent 53131A universal counter. A QCM with fundamental frequency of 5 MHz with frequency shifts of 20 Hz was reported in [3], and the measurements were performed using QCM200 Quartz Crystal Microbalance (Stanford Research Systems). 16 MHz QCM with frequency shift close to 20 Hz was reported in [4].

Accuracy of most frequency measurement techniques is limited by measurement time [5], [6]. As consequence if more precision is required, longer times for measuring are needed. A novel technique proposed by Sergiyenko-Hernandez [7] is a powerful tool for fast frequency measurement. Advantages of this method have been explored, such as invariance to jitter [8] and high resolution [9]. Also, diverse applications on automotive [10], [11] and aerospace industries [12], [13] have been proposed. The frequency measurement principle by rational approximations [7] is a suitable tool for analyzing frequency shifts in FDS outputs. Most analysis are made when the measurand is in steady-state, this means that frequency value has a fixed value. In this work, frequency shift analysis is introduced, and results of analysis are presented and discussed.

II. Frequency measurements using the rational approximations principle

In this section, a short review of the frequency measurement principle by rational approximations is introduced. Also the mathematical modeling and parameters of signals required for measuring is presented.

The frequency measurement principle by rational approximations states that, by comparing a signal to be measured (S_x), having an unknown frequency (f_x) against a reference signal (S_0) whose frequency (f_0) is known, f_x can be calculated. In this method, both signals must be conditioned in such way, that the pulse width (τ) in each period in both signals is the same, also $\tau < T_0$, where T_0 is the period of S_0. Pulses in signals after conditioning process are shown on Fig. 1, where t_r, t_h, t_f,t_l, are falling time, high level time, falling time, and low level time respectively. V_h is the high level voltage, and V_l is the low voltage level .

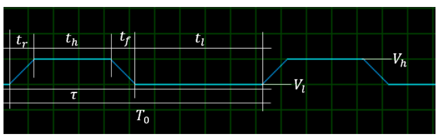

Figure 1. Pulses in signals after signal conditioning

After the previous conditions are fulfilled, the signals are multiplied using the AND-logical function, this leads to the creation of a coincident pulse train ($S_x \& S_0$ of Fig. 2). When the pulses in both signals exist at the same time, there is a pulse coincidence, if the duration of the coincidence has the same value than τ, a perfect coincidence exists, otherwise there is a partial coincidence. As it can be seen on Fig. 1, the pulses in any signal ($S_x \& S_0$) are not perfectly square. The last is due the existence of t_r and t_f, as consequence the ideal value of τ is $\tau = t_r + t_f + t_h$. If $\tau >> t_r$ and $\tau >> t_f$, then $\tau \approx t_h$. The true value of τ is of paramount importance in real systems. In [13], [12], coincidence detectors were proposed. The variations of between V_l and V_h affect the duration of τ, because the digital circuits used for building a coincidence detector are limited in their input by a low-high logic level. The true value of τ is $t_h < \tau < t_r + t_f + t_h$.

When two perfect coincidences exist, a mediant fraction is formed, and the condition of Eq. 1 es hold (Fig. 2):

$$\frac{P_{n+i}}{Q_{n+i}} < \frac{P_{n+i} + P_n}{Q_{n+i} + Q_n} < \frac{P_n}{Q_n}, \quad (1)$$

where, P_n and Q_n are independent counts of unknown and reference pulses on the considered interval of time (as shown on Fig. 2), n is the number of fraction and i is an entire positive number; speaking precisely, P_{n+i} and Q_{n+i} appear after one iteration after P_n and Q_n, the $n+i$ value increases as result of time measurement until a number m where the best

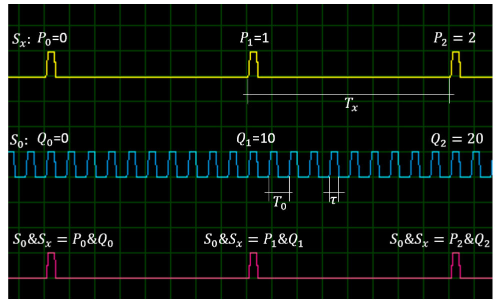

Figure 2. Frequency measurement by rational approximations: signals comparison

approximation is obtained, this is another well known property in classic metrology, by increasing measurement time, the accuracy of measurement increases [14]. Finally the unknown frequency value (f_x) of S_x can be obtained using the known frequency value (f_0) of standard S_0:

$$f_x = f_0 \frac{P_n}{Q_n}, \quad (2)$$

from Eq. 1 and 2:

$$f_x = f_0 \frac{\sum_m P_n}{\sum_m Q_n}. \quad (3)$$

The relative error (β) for the frequency measurement principle by rational approximations is given by:

$$\beta = \frac{\left| f_x - \frac{P_n}{Q_n} f_0 \right|}{f_x}, \quad (4)$$

and the measurement time (M_t) since the beginning of measurement process until the $n-$fraction is expressed as:

$$M_t = Q_n T_0 = \frac{Q_n}{f_0}. \quad (5)$$

III. Measurement model simulation

The mathematical model exposed in section 2 can be analyzed using computational tools. The model a frequency measurement system using rational approximations has been previously explored, and analyzed trough computational simulations [1], [7], [12], [9], [15]. In the cited papers, just stationary frequency values were analyzed. For the purposes of this work, new simulations algorithms are proposed and implemented in Matlab.

Given the simulation time ($SimTime$) and the period (T) of a signal to model, an array of numbers (integers or floating values) is generated using the algorithm 1. In each iteration, the array time keeps an addition of the signal's periods. The number of periods correspond to the number of iterations (i). Also in each iteration, the value of $ElapsedTime$ increments by one value of T. The array $Time$ keeps increasing while the $ElapsedTime$ is lower than $SimTime$.

Our interest is to detect when a coincidence occurs. According to Fig. 2, when a coincidence occurs a time of coincidence

Algorithm 1 Signal generator

SIGNAL(T, $SimTime$)
1 $ElapsedTime = i = 0$
2 while $ElapsedTime < SimTime$
3 $Time[i] = ElapsedTime$
4 $ElapsedTime = Time[i] + T$
5 $i++$
6 return$Time$

(t_c) exists. Our analysis indicates that $0 < t_c \leq \tau$. The algorithm 2 allow us to get two arrays (P, Q), that correspond to the fraction where the coincidence occurred. Algorithm 2 works as follows. In line 1, the variables used for counting are initialized. Unknown frequency and reference signals are generated using algorithm 1 in lines 2 and 3 respectively. Two "while" loops are used for comparing data of each position in $Signalfo$ and $Signalfx$. In line 6 is calculated the time of coincidence. If the condition on line 7 holds, the values of the fraction where the coincidence occurred are recorded.

Algorithm 2 Coincidence detector

COINCIDENCE($SimTime$, τ, T_x, T_0)
1 $a, b, c = 0$
2 $Signalfo$ =SIGNAL(T_0, $SimTime$)
3 $Signalfx$ =SIGNAL(T_x, $SimTime$)
4 while $a < length(Signalfo)$
5 while $b < length(Signalfx)$
6 $t_c = \tau - |Signalfo[a] - Signalfx[b]|$
7 if $t_c > 0$
8 $P[c] = b$
9 $Q[c] = a$
10 $c++$
11 $b++$
12 $a++$
13 return P, Q

The values of P_n, Q_n obtained trough algorithm 2 can be used for calculating the value of f_x (Eq. 3) or relative error (Eq. 4).

In the next section, the presented algorithms are implemented and used for frequency measurement.

IV. Measuring frequency shifts in FDS

In a FDS before a measurand change, there is an initial frequency value (f_s), and after change a different frequency (f_p) value exists. After measurement, the value of f_x can either approximate to f_s or f_p.

A frequency shift (Δf) is defined as:

$$\Delta f = f_s - f_p. \quad (6)$$

According to Eq. 6, the value of a frequency shift could be obtained by measuring f_s before, and f_p after measurand change. From Eq. 6, new operation conditions are needed for frequency meters. If the value of f_x changes, it will affect the values of P_n and Q_n in Eq. 2 and 3. In this section, we will discuss the effect of frequency shifts in fast frequency measurement using the rational approximations principle.

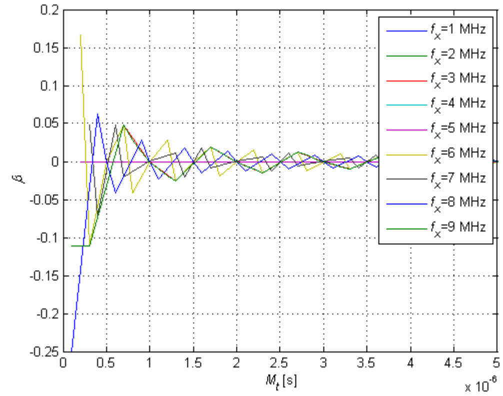

Figure 3. Relative error in frequency measurement process using $f_0 = 10$ MHz

A. Measurement of f_s

In previous works, the rational approximation principle has been used for measurement of steady frequency values [12], [13]. The output of a FDS has an unknown frequency value, such value can be modeled using the previously proposed algorithms. Let us consider $f_s = f_x$, $f_x = \{1, 2, ..., 9\}$ MHz, $\tau = \frac{4}{10}T_0$, and $T_0 = 10$ MHz. The frequency measurement process is simulated using algorithm 2, and the relative error of the analyzed values of f_x is shown on Fig. 3

The relative error (β) is calculated using Eq. 4. The data obtained from simulations is summarized on Table I. As it is shown in Fig. 3, for all the analyzed values, β has a maximum relative error in the two first fractions This is due to the first fraction that has $P_0 = Q_0 = 0$, and the next fraction is the first approximation. After P_1 and Q_1, β decreases while the measurement time is incremented. As it has been shown in previous works, if the principle of rational approximations is used for frequency measurement, the value of β reduces when the measurement time increases (as it can be seen on Table I and using Eq. 5).

Also, a very important property of the rational approximations principle is shown: by increasing the order of f_0 a better approximation to f_x is obtained. For this case, when $f_0 = 10$ MHz (Fig. 3), the value of f_x is approximated in a longer time than when $f_0 = 100$ MHz (Fig. 4). If $f_0 = 100$ MHz, the best approximation to f_x is obtained almost since the start of frequency measurement. The data obtained from simulations is shown on Table 4.

There are cases where $P_1/Q_1 = f_x/f_0$, these are known as perfect coincidences. In consequence:

$$\frac{nP_1}{nQ_1} = \frac{f_x}{f_0}, \quad (7)$$

the last will be the best approximations since $n = 1$. In Table I, there are perfect coincidences since $n = 1$ when $f_x = 1$, 2, 4 and 5. From Table I, a relationship between f_x, f_0 and

Table I
DATA OBTAINED FROM COMPUTATIONAL EXPERIMENTS WHEN $f_0 = 10$ MHz

f_x [MHz]	$\frac{P_1}{Q_1}$	β in $n=1$	$\frac{P_m}{Q_m}$	β in m	$\frac{\sum_m P_n}{\sum_m Q_n}$
1	$\frac{1}{10}$	0	$\frac{9999}{99990}$	0	$\frac{49995000}{499950000}$
2	$\frac{1}{5}$	0	$\frac{19999}{99995}$	0	$\frac{199990000}{999950000}$
3	$\frac{1}{3}$	-0.1111	$\frac{29999}{99997}$	3.33×10^{-6}	$\frac{449985000}{1.49995\times10^{9}}$
4	$\frac{2}{5}$	0	$\frac{39998}{99995}$	0	$\frac{399980000}{999950000}$
5	$\frac{1}{2}$	0	$\frac{49999}{99998}$	0	$\frac{1.249975\times10^{9}}{2.49995\times10^{9}}$
6	$\frac{1}{2}$	0.1667	$\frac{59999}{99998}$	-3.33×10^{-6}	$\frac{1.79997\times10^{9}}{2.99995\times10^{9}}$
7	$\frac{2}{3}$	0.0476	$\frac{69998}{99997}$	-1.42×10^{-6}	$\frac{1.749965\times10^{9}}{2.49995\times10^{9}}$
8	$\frac{1}{1}$	0.2500	$\frac{79999}{99999}$	2.5×10^{-6}	$\frac{2.39996\times10^{9}}{2.99995\times10^{9}}$
9	$\frac{1}{1}$	-0.1111	$\frac{89999}{99999}$	1.11×10^{-6}	$\frac{3.149955\times10^{9}}{3.49995\times10^{9}}$

Table II
DATA OBTAINED FROM COMPUTATIONAL EXPERIMENTS WHEN $f_0 = 100$ MHz

f_x [MHz]	$\frac{P_1}{Q_1}$	β in $n=1$	$\frac{P_m}{Q_m}$	β in m	$\frac{\sum_m P_n}{\sum_m Q_n}$
1	$\frac{1}{100}$	0	$\frac{9999}{999900}$	0	$\frac{49995000}{4.9995\times10^{9}}$
2	$\frac{1}{50}$	0	$\frac{19999}{999950}$	0	$\frac{199990000}{9.9995\times10^{9}}$
3	$\frac{1}{33}$	-0.0101	$\frac{29999}{999967}$	3.33×10^{-7}	$\frac{449985000}{1.49995\times10^{10}}$
4	$\frac{1}{25}$	0	$\frac{39999}{999975}$	0	$\frac{799980000}{1.99995\times10^{10}}$
5	$\frac{1}{20}$	0	$\frac{49999}{999980}$	0	$\frac{1.249975\times10^{9}}{2.49995\times10^{10}}$
6	$\frac{1}{17}$	0.0196	$\frac{59999}{99998}$	-3.33×10^{-7}	$\frac{1.79997\times10^{9}}{2.99995\times10^{10}}$
7	$\frac{1}{14}$	-0.0204	$\frac{69999}{999986}$	2.857×10^{-7}	$\frac{1.749965\times10^{9}}{2.49995\times10^{9}}$
8	$\frac{2}{25}$	0	$\frac{79998}{999975}$	0	$\frac{1.59996\times10^{9}}{1.99995\times10^{10}}$
9	$\frac{1}{11}$	-0.1111	$\frac{89999}{999989}$	1.11×10^{-6}	$\frac{3.149955\times10^{9}}{3.49995\times10^{10}}$

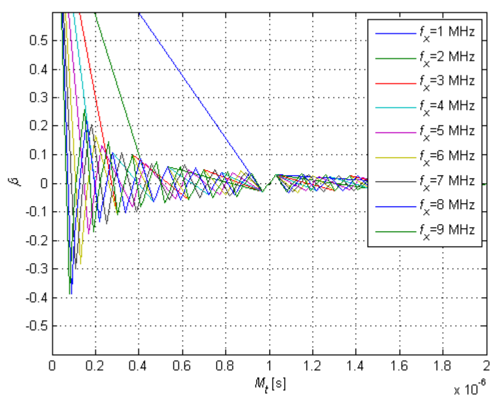

Figure 4. Relative error in frequency measurement process using $f_0 = 100$ MHz

β can be inferred. When the value of f_x approaches to f_0, the relative error is incremented. Also P_n, Q_n start in a value closer to 1,until a best approximation is obtained when M_t increases. The last observations, lead us to the hypothesis: "if the ratio f_0/f_x is greater, the value of Q_n will increase faster. As consequence, the best approximation could be obtained in shorter time".

In Fig. 4, the results of computational experiments using $f_0 = 100$ MHz are shown. The other parameters used for simulation still the same used in our previous analysis.

By comparing Fig. 3 and 4, an observation can be done. In the last, the time needed for achieving a better measurement is lower than in Fig. 3. According to Eq. 5, the measurement time decreases when f_0 is increased. In consequence, even if there are the same values of P_n and Q_n, the best coincidences will appear in a shorter time. The data shown in Fig. 4 is summarized in Table II.

A comparison of data shown in Table I and II highlights the importance of the relationship between f_x and f_0. In Table II, the value of Q_n increases faster than in Table I. This is due more pulses of S_0 are counted until the next coincidence appears. Another observation is noted, when the value of f_x increases, the value of P_n/Q_n approaches to f_x/f_0 faster. Our analysis is bound to a simulation time (algorithm 1, 2). If the time used for analysis is increases, grater values of P_n/Q_n would appear and a better approximation to f_x/f_0 could exist. As result, the relative error in frequency measurement decreases. So far, our hypothesis is held and validated by the results of computational experiments.

The resolution of our frequency measurement process is defined by n and f_0. When n increases, more coincidences exist and the value of P_n/Q_n increases. If f_0 has more orders of magnitude than f_x, the value of Q_n increases faster. By comparing Table 4, smaller relative error and better resolution are shown.

Further analysis shows that by increasing the order of f_0, better approximations in shorter time can be obtained. For practical purposes, there are few constrains. Such as stability of reference standard, and capacity of the circuits used for detecting a pulse width as short as $\tau < T_0/2$

B. *Measurement of f_p*

Before we start our analysis, we should make a remark. The frequency measurement principle by rational approximations is based on number theory and deals only with entire numbers [16]. A mediant fraction (Eq. 1) is also known as continuous fraction. The rational approximations are formed by coincidence of pulses, this implies the fact that only complete pulses are counted. Existence of perfect and/or partial coincidences, only depends of the apparition of pulses during a given time.

According to Eq. 6, if $f_x = f_p$ a frequency shift exists. From our models, the time where the best approximation appears is very short. In order to shown a a perceptible change,

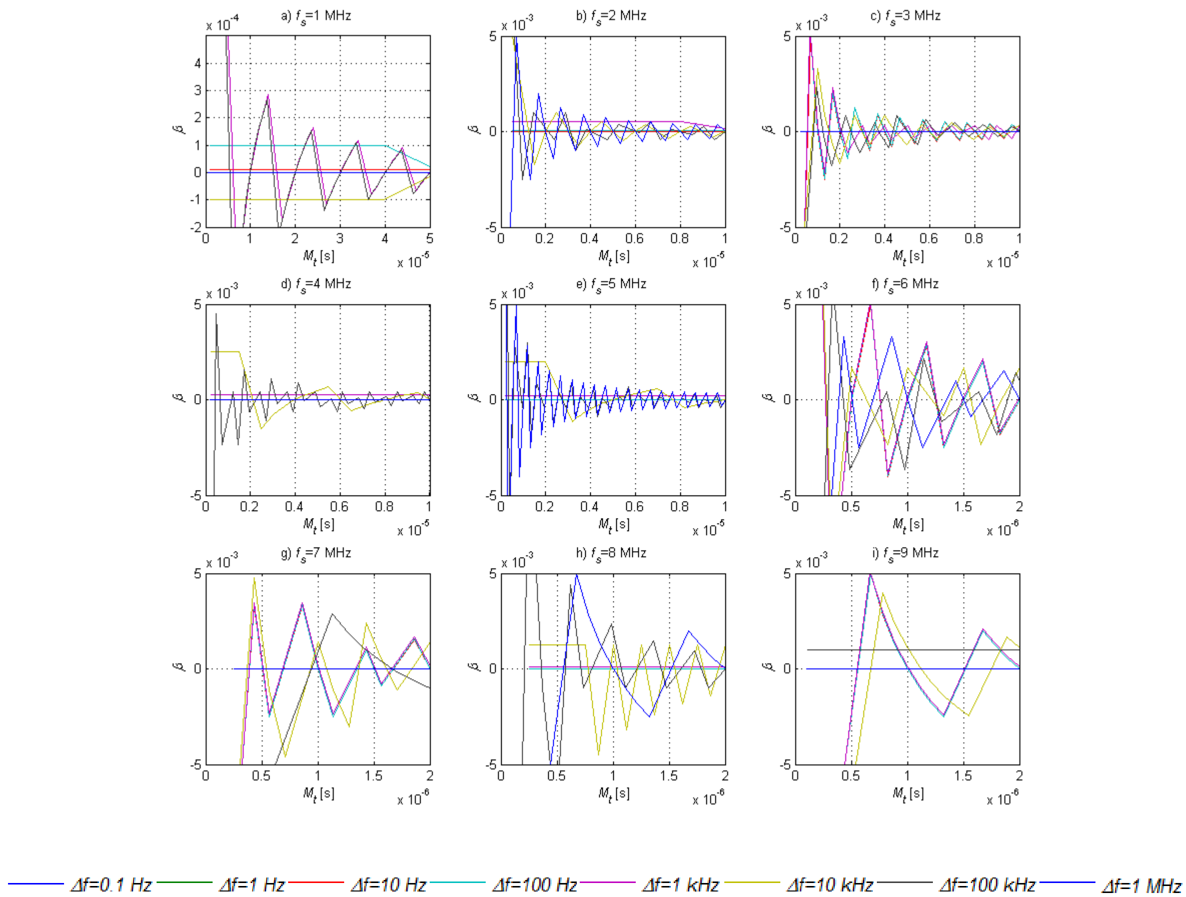

Figure 5. Relative error (β) during measurement time (M_t) in frequency measurement process of $f_s + \Delta f$: a) $f_s = 1$ MHz, b) 2 MHz, c) $f_s = 3$ MHz, d) $f_s = 4$ MHz, e) $f_s = 5$ MHz, f) $f_s = 6$ MHz, g) $f_s = 7$ MHz, h) $f_s = 8$ MHz, i) $f_s = 9$ MHz.

all the physical variables need a minimum time. Under the idea that our measurement is faster than the speed of change of measurand, the measurement of f_p can be conceived as stationary state process. From Eq. 3 and 6, the frequency shift can be calculated as:

$$\Delta f = f_0 \frac{\sum_m P_n}{\sum_m Q_n} - f_s \tag{8}$$

Using algorithms 1 and 2 several simulations were carried on. The purpose of such computational experiments was to get clarity about the smallest perceptible Δf that could be measured. The simulations were carried on with Δf={0.1, 1, 100, $1{\times}10^3$, $1{\times}10^4$, $1{\times}10^5$, $1{\times}10^6$} Hz, $f_p = f_x$, $f_x = \{1, 2, ..., 9\}$ MHz, $\tau = \frac{4}{10}T_0$, and $f_0 = 100$ MHz. The results are shown on Fig. 5.

After enough time, the relative error shown in Fig. 5 has the same behavior of experiments shown in Fig. 3 and Table II. Also in Fig. 5, in order to appreciate the influence of Δf a smaller measurement time is shown.

In particular, the Fig. 5 b,c,d,e have a similar behavior during the same measurement time. If Δf increases, a smaller influence on β occurs. When the value of f_x approximates to f_0, β decreases very fast (almost since the beginning of measurement process) or last some time to converge to a near zero value. If f_x and f_0 are close, the number of coincidences is incremented. When Δf decreases, β is less affected, until β is closer as possible to zero. The worst cases are when $f_s = 1$ MHz and $f_s = 9$ MHz, this is because $f_s + \Delta f$ is either very close or very far from to f_0.

In general, when the value of f_x approaches to f_0, β needs more time for decreasing. Also, Δf is a small approximation of f_x to f_0. This allow us to think that such approximation increments the measurement time required for getting the smallest value possible of β. From previous analysis we learned, that if the value of f_x is close to f_0 a higher error exists. In the data shown in Fig. 5, in some cases small values of β exists in very short time. The last statements, allow us to think it is because a partial coincidence exist where:

$$\Delta f + f_x = f_x + \beta f_x, \tag{9}$$

or:

$$\Delta f \approx \beta f_x. \tag{10}$$

Eq. 10 has sense only when n has a small value and $P_1/Q_1 \approx \sum_m P_n/\sum_m Q_n$. This indicates that β does not decrease. On the other hand, the best approximations are obtained when:

$$P_1/Q_1 << \sum_m P_n/\sum_m Q_n \tag{11}$$

or

$$P_1/Q_1 >> \sum_m P_n/\sum_m Q_n \tag{12}$$

and $\beta \approx 0$.

For each case analyzed on Fig. 5, there is an increment in the number of coincidences. This es the result of a increment of the ratio P_n/Q_n.

Clearly, our computational experiments have shown there is a relationship between f_x, f_0, n and Δf. The smallest detectable value of Δf is bounded by Eq. 10.

V. Conclusions and future work

In this work, by using the rational approximations principle, an initial approach to the problem of measuring frequency shifts on FDS has been carried on. We have shown how accuracy of frequency measurements is affected for many factors. Such as variations in the values of f_x, f_0 and Δf.

Many analysis were carried on using the proposed algorithms. In the future we hope to study other properties of our measurement method. Also an ongoing work includes a comparison of algorithms 1 and 2 with a real proposed system [12].

One of the principal contributions of this work is that when f_0 is incremented, a higher resolution can be achieved. The limitations for real systems are stability of reference standard, and capacity of the circuits used for detecting a pulse width as short as $\tau < T_0/2$.

Finally, a bound for the smallest detectable value of Δf is given by Eq. 10.

As future work, we expect to increase our analysis and improve the resolution of our measurement technique. This, with the aim of detecting smaller values of Δf.

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