Rank filtering with adaptive neighborhoods

Vitaly Kober and Josué Alvarez-Borrego

Optics Department, Division of Applied Physics, CICESE
Km 107 Carretera Tijuana-Ensenada, Ensenada 22860, B.C., México

ABSTRACT

A new approach to design rank-order filters based on an explicit use of spatial relations between image elements is proposed. Many rank-order processing techniques may be implemented by applying the approach, such as noise suppression, local contrast enhancement, and local detail extraction. The performance of the proposed rank-order filters for suppression a strong impulsive noise in a test interferogram-like image is compared to the median filter. The comparisons are made using a mean square error, a mean absolute error, and a subjective human visual error criteria.

Keywords: Rank-order filters, median filter, nonlinear adaptive filtering, noise smoothing.

1. INTRODUCTION

In recent years the use of nonlinear filters based on the calculation of rank-order statistics has been increasing in computer and optical research. There are several different classes of nonlinear filters, which incorporate rank-order operations in one way or another. A particular list of the filters would include median filters,\(^1\) multistage and multilevel median filters, stack filters, alpha-trimmed mean filters, order statistics filters, morphological filters, rank-order filters.\(^2,3\) These filters have all proven to be very effective in removal of additive and impulsive noise, enhancing and restoring images. Moreover, they exhibit excellent robustness properties and provide solutions in many cases where linear filters are inappropriate. Perhaps the primary reason for their success in image processing is that they can suppress noise without destroying important image details such as edges and fine lines. A drawback of conventional rank-order filters is that they weakly exploit spatial relations between image elements, because they perform the reordering of elements of a two-dimensional moving window into a one-dimensional sequence. We propose to design rank-order filters with the use of spatial relations between image elements. The filters utilize spatial and rank information of the input image within a moving window to produce the output. The output of the proposed filters is a function over spatially connected elements of the variational row of a moving window.

2. RANK-ORDER FILTERS

The rank filtering is a locally adaptive processing of the signal in a moving window. First, using different neighborhoods, we define desirable structures in the window. Next, the estimation approach can be applied to the elements of the neighborhood structures to compute an estimate of the central pixel of the window with respect to different criteria. Let us introduce some useful notation and definitions: \(s(n,m)\) is a vector of pixels of the image to be processed, that has \(Q\) gray-scale levels of quantization; \(n, m\) are coordinates of the pixels, \(n=1,2,\ldots,N\) and \(m=1,2,\ldots,M\); \(L=N\times M\) is the image matrix size; \(v = \{v_{n,m}\}\) is a vector of pixels of the noise-free (original) image; \(\hat{v} = \{\hat{v}_{n,m}\}\) is a vector of pixels of the resulting image.

The spatial neighborhood (\(S\)-neighborhood) for each image pixel can be defined as a set of pixels surrounding the given one geometrically. An important notion in order statistics is the variational row, which is defined as a one-dimensional sequence \(\{V(r)\}\) of \(K\) pixels whose elements are sorted in ascending order with respect to their values: \(\{V(r): V(r) \leq V(r+1), r=1,2,\ldots,K\}\). Here \(V(r)\) and \(r(V)\) are called the \(r\)th order statistics and the rank of the value \(V\), respectively. All the parameters of rank-order filters are functions of local, or short time, histograms computed over pixels of the spatial neighborhoods. Therefore, the computational complexity of the rank processing depends on calculation of local histograms. To describe different structures in the image, we define the following subsets over either the \(S\)-neighborhood: \(^2,3\)

- **EV-neighborhood** is a subset of pixels \(\{v_{n,m}\}\) whose values deviate from the value of the central pixel \(v_{k,l}\) at most by predetermined quantities \(\pm \varepsilon\). \(K\)-neighborhood is defined as a subset of a specified number \(K\) of pixels \(\{v_{n,m}\}\) whose values are nearest to the value of the central pixel \(v_{k,l}\). A subset of pixels whose ranks deviate from that of the central pixel at most by predetermined quantities \(\pm \varepsilon\) is called the **ER-neighborhood**.

The choice of neighborhood (\(NBH\)) is defined by the available a priori information on the processed image. For example, if

Further author information – V.K.(correspondence): Emails: vkober@cicese.mx, vkober@hotmail.com; Telephone: +52-6-1745050; Fax: +52-6-1750553, J.A.-B.: Email: josue@cicese.mx
a priori information about the geometrical size \( K \) of the details to be preserved is known then the \( KNV \)-neighborhood can be used. The parameter \( K \) is chosen of the order of the detail area to be preserved after further processing. The choice of the \( EV \)-neighborhood helps us to take into account a priori information about either the spread of the signal to be preserved or noise fluctuation to be suppressed. The \( ER \)-neighborhood is often used in edge extraction algorithms and for suppression of a mixture of additive gaussian-type noise and noise with a distribution having heavy tails. The size of the \( ER \)-neighborhood is determined by the part of the outliers in the distribution. Finally, note that the size of the \( S \)-neighborhood should be of the order of the double size of the minimal structure to be preserved after processing. Three types of estimation from the theory of robust estimation of location parameters can be utilized to compute an estimate of the central pixel of the neighborhoods, namely the L-estimator based on linear combination of order statistics, the R-estimator derived from rank tests; and the M-estimator or the maximum likelihood estimator. All the types of estimation can be implemented using a few basic operations over the introduced neighborhoods. The operations are defined as follows: \( SIZE(NBH) \) is the quantity of pixels forming the neighborhood, \( MEAN(NBH) \) is the sample mean over the neighborhood, \( MED(NBH) \) is the median value over the neighborhood, \( MIN(NBH) \) is the minimum over the neighborhood, \( MAX(NBH) \) is the maximum over the neighborhood, \( CUT(NBH) \) is cross-cut through the neighborhood, In computer experiments we use an efficient algorithm for suppression of mixed additive and impulsive noise:

\[
\hat{v}_{i+1}^{m,n} = \begin{cases} 
\text{MEAN}\left( \left. EV \left( v_{i,n}^{m} \right) \right\} \right) & \text{if } SIZE(EV(\left. v_{i,n}^{m} \right\} ) \geq \text{Thrd}^d \\
\text{MED}(S(\left. v_{i,n}^{m} \right\} ) - EV(\left. v_{i,n}^{m} \right\} )) & \text{otherwise}
\end{cases}
\]

where \( \text{Thrd}^d \) is a threshold value of outlier detection at the \( d \)th iteration, \( - \) denotes the set difference operation. The algorithm results either in the sample mean of the pixels of the \( EV \)-neighborhood, or in the median value of pixels of the \( S \)-neighborhood.

3. RANK-ORDER FILTERS USING SPATIAL CONNECTIVITY OF PIXELS

The spatial nearness does not always form spatial clusters. For example, the pixels constituting neighborhoods as defined in Section 2 are not necessary spatially connected to the central pixel for which the neighborhood was formed. Therefore, one can conclude that conventional rank-order filters designed with the use of one-dimensional local histograms weakly exploit spatial relations between image elements. To overcome this drawback, we supplement the neighborhood definitions by requiring that all the pixels of the used neighborhood should be spatially connected of each other. We say, (a) two different pixels \( v_{k,l} \) and \( v_{m,n} \) are spatial neighbors if their coordinates satisfy the following conditions: \( |k-m| + |l-n| = \Delta \), where \( \Delta \) is a positive constant, which is called the order of connectivity; (b) path from pixel \( v_{k,l} \) to pixel \( v_{m,n} \) is a sequence of the pixels \( A_1, A_2, ..., A_h \) of a neighborhood where \( A_1 = v_{k,l} \), \( A_h = v_{m,n} \), and \( A_{i+1} \) is a spatial neighbor of \( A_i \) and \( i = 1, ..., h-1 \); (c) two pixels are called spatially connected if there is a path between these pixels in a neighborhood. The parameter \( \Delta \) is well suited to describe connected regions of images corrupted with impulsive noise. Using the definitions, we introduce the concept of adaptive neighborhood (ANBH). The size and shape of adaptive neighborhood are dependent on characteristics of image data and on parameters, which define measures of homogeneity of pixel sets. New connected sets are adaptive neighborhoods, which are referred to as the \( AEV \), \( AER \), \( AKNV \)-neighborhoods, respectively. The output of filtering is a value computed as the basic operations \( MEAN(ANBH) \), \( MED(ANBH) \), \( MIN(ANBH) \), \( MAX(ANBH) \), \( CUT(ANBH) \) at all possible pixels in the adaptive neighborhood. The operations may be iteratively applied several times.

4. COMPUTER EXPERIMENTS

Computer experiments are carried out to illustrate and compare the performance of conventional and proposed algorithms. We base our comparisons on the mean square error (MSE), the mean absolute error (MAE), and a subjective visual criterion. In our computer experiments, a test image has 512x512 pixels, and each pixel has 256 levels of quantization. An enhanced difference visual display is used to quantify the error in a human visual error criterion. If there is no error between the original image and the filtered image, at a pixel location, this pixel is displayed as gray. For a maximum error, the pixel is displayed either black or white. We compare the following algorithms. MED is a square median filter of size 3x3 elements. RA is a rank algorithm given in Eq.(1), which uses the \( AEV \)-neighborhood with the order of connectivity of 2 instead of the \( EV \)-neighborhood. The number of iterations is 6. The size of the moving window is always 21x21. The size of the \( S \)-neighborhood for impulsive noise removal is 5x5. The threshold values are 2, 4, 6, 8, 10, 11.

Fig. 1(a, b) shows the original test image and the image corrupted with impulsive noise. The probability of impulse occurring is 0.2, and if it occurs it can be positive or negative with equal probability. In the simulations, the values of the impulse were set to 0 or 255. Table 1 shows the difference between the original and noisy images in terms of the MSE and the MAE. Fig. 1(c, e) shows the processed images with (c) MED, (e) RA. Fig.1(d, f) shows an enhanced difference of the original image with (d) the median filtered image, (f) the filtered image with RA. Table 1 shows the error retained and introduced by each of the filters under the MSE and MAE error criteria. It can be clearly seen that the proposed algorithms outperform the median filter.
Table 1. Results of impulsive noise suppression with different filters.

<table>
<thead>
<tr>
<th>Type of Filters</th>
<th>Measured Errors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MSE</td>
</tr>
<tr>
<td>Noisy image</td>
<td>0.2196</td>
</tr>
<tr>
<td>Median 3x3</td>
<td>0.0898</td>
</tr>
<tr>
<td>RA</td>
<td>0.0357</td>
</tr>
</tbody>
</table>

Fig. 1. (a) Original test image, (b) noisy image (impulsive noise). (c) median filter, (d) enhanced difference between original image and median filter., (e) RA, (f) enhanced difference between original image and RA.

REFERENCES