Noise analysis of two pattern recognition methodologies using binary masks based on the fractional Fourier transform

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ABSTRACT

Noise often corrupts images; therefore, it is essential to know the performance capability of a pattern recognition algorithm for images affected by it. In this work, a complete analysis of two methodologies is performed when images are affected by Gaussian and salt and pepper noise. The two methods use the nonlinear correlation of signatures. A signature is a one-dimensional vector that represents each image, and it is obtained using a binary mask created based on the fractional Fourier transform (FRFT). In the first methodology, a spectral image it is used as the input to the system. The spectral image is the modulus of the Fourier transform (FT) of the image processed. The binary mask is generated from the real part of the FRFT of the spectral image. The signature is constructed by sampling the modulus of the FRFT of the spectral image with the mask. In the second methodology, the image is the input to the system, and the binary mask is obtained from the real part of the FRFT of the image. The signature, in this case, is obtained by sampling the modulus of the FT of the image with the binary mask. Each method was tested using the discrimination coefficient metric.

Keywords: Image processing, pattern recognition, fractional Fourier transform, binary ring masks, Fourier optics

1. INTRODUCTION

Images affected by noise is an important problem in pattern recognition. Thus it is necessary to develop new techniques capable of dealing with this images. Recently, systems based on binary ring masks obtained from the FT\(^1,2,3\), scale transform\(^4\) and Bessel masks\(^5\) have been developed. These methods are invariant to position and rotation and scale. Also, they respond very well to images affected by noise. This work presents two pattern recognition systems based on signatures generated with a binary mask obtained by using the real part of the FRFT. The FRFT allow to select and work between the space plane and the Fourier plane to create the binary rings mask and get better discrimination. Also, for the discrimination between signatures, nonlinear correlation is implemented.

The FRFT is a generalization of the FT that depends on a parameter \(\alpha\) and is defined by Almeida\(^6\) as

\[
G_\alpha(u) = \mathcal{F}_\alpha \{ g(x) \} = \sqrt{\frac{1-j\cot(\alpha)}{2\pi}} e^{\frac{x^2}{2\cot(\alpha)}} \int_{-\infty}^{\infty} g(x) e^{\frac{jx^2}{2\cot(\alpha)}} e^{-j\alpha \pi x^2} dx, \tag{1}
\]

where \(j = \sqrt{-1}\), \(x\) is the variable of the original function in the space domain, \(u\) is a dummy variable and \(\alpha\) is the rotation angle in the space-frequency plane. Defining \(\mathcal{F}\) as the FT operator, the FRFT of the order \(p\) of a function \(g(x)\) is \(\mathcal{F}^p \{ g(x) \} = G_\alpha(u)\), where\(^7\)

\[
p = \frac{\alpha}{\pi} \cdot \tag{2}
\]

Equation (2) shows that by changing the \(\alpha\) angle, a fractional order \(p\) is achieved. Therefore the classical FT operator is a particular case of the FRFT of order \(p = 1\) or equivalently, an angle \(\alpha = \pi / 2\). In this work, the FRFT is calculated based on the \(\alpha\) angle. For simplicity, the angle is taken in degrees rather than radians. Figure 1 shows the FRFT applied to the image of diatom species Actino cyclycus in geste Rattray at different \(\alpha\) values, going from the space domain in Fig. 1a to the frequency domain Fig. 1f.
One important property is that the modulus of the FRFT is not invariant to translation as in the case of the modulus of the FT\textsuperscript{3,7}. Therefore two different methodologies to achieve invariance to translation were developed.

2. METHODOLOGIES.

The methods work with $N \times N$ gray-scale images only.

2.1 Methodology 1. Working with the FRFT of the spectral image to create the binary rings mask

For an image $I(x,y)$, where $x = 1,\ldots,N$, $y = 1,\ldots,N$, the spectral image is the modulus of its FT:

$$I_s = |\mathcal{F}\{I(x,y)\}|.$$  \hspace{1cm} (3)

By working with the spectral image, position invariance is achieved. In Fig. 2a we have an $256 \times 256$ image of diatom species Actinocyclus inges Rattray. Figure 2b shows the spectral image taken from Fig. 2a.
2.1.1 The $M_{RN}$ binary rings mask

The binary mask is built using the real part of the FRFT of the spectral image: $\text{Re}\left[\mathcal{F}_\alpha \{I_s\}\right]$. Figure 2c shows the real part of the FRFT to an angle $\alpha = 85^\circ$ applied to Fig. 2b.

Then, the real part of the FRFT of a $N \times N$ pixels image is filtered with a disk mask $D$, defined as

$$D = \begin{cases} 1, & \text{if } d\left((c_x,c_y),(x,y)\right) \leq \frac{N}{2}, \\ 0, & \text{otherwise}, \end{cases}$$

(4)

where $(c_x,c_y)$ is the center pixel, $N$ is the diameter, and $d\left((c_x,c_y),(x,y)\right)$ is the Euclidean distance between each pixel and the central pixel.

Figure 3. Filtering process of the real part of the FRFT of the spectral image by a unitary disk.
The filtering process is given by

\[ f_{Re} = D \cdot \text{Re} \left( \mathcal{F}_u \{ I_x \} \right), \tag{5} \]

where \( \bullet \) is a point-to-point product. Fig. 3 shows the filtering process.

Next, from \( f_{Re} \) 180 profiles with length \( N \) are obtained with a separation of one degree passing through the center pixel \( (c_x, c_y) \). Figure 4a show some example profiles from the \( f_{Re} \).

Figure 4. (a) Some examples of the 180 profiles obtained from the \( f_{Re} \). (b) Optimum profile selected from the \( f_{Re} \).

The profile equation is

\[ P_{Re}^0(x) = f_{Re} \left( x, y(x) \right), \tag{6} \]

where \( y(x) = m(x - x_1) + y_1 \), \( m \) is the slope of \( y \), \( (x_1, y_1) = (c_x + c_x \cos \theta, c_y - c_y \sin \theta) \) and \( (x_2, y_2) = (c_x + c_x \cos (\theta + \pi), c_y - c_y \sin (\theta + \pi)) \) are the endpoints of the line, and \( \theta \) is the angle between \( y \) and the horizontal axis.

Then, the energy in each profile is calculated:

\[ S_{Re}^0 = \sum_{x=1}^{N} P_{Re}^0(x)^2, \tag{7} \]

and the profile with the maximum energy will be selected.
where $\beta$ and $\gamma$ are the angles of the profiles that have the maximum energy and $Op_{Re}(x)$ is the optimum profile from the real part of the FRFT, shown in Fig. 4b.

Next, a binary function $Z_{RN}(n_x)$ is built from the optimum profile by

$$Z_{RN}(n_x) = \begin{cases} 1, & \text{if } m_{Re}(n_x) < 0, \\ 0, & \text{if } m_{Re}(n_x) \geq 0. \end{cases}$$

(9)

where $m_{Re}(n_x)$ is the slope of the optimum profile and $n_x = 1, \ldots, N-1$.

Then taking $n_x = 1$ as the rotation axis, the binary function is rotated $360^\circ$ to obtain concentric cylinders of height one. Because the entire function $Z_{RN}(n_x)$ is rotated, to get a mask of a size no larger than the size of the original image, the radius of the cylinders are divided by two. The binary mask is named $M_{RN}$. Figure 5 shows the binary masks obtained at different $\alpha$ angles of the FRFT for the spectral image of diatom species *Actinocyclus inges Rattray*.

![Binary masks](image)

Figure 5. $M_{RN}$ mask for diatom species *Actinocyclus inges Rattray* using method 1 for different $\alpha$ values. (a) $\alpha = 85^\circ$. (b) $\alpha = 45^\circ$.

2.1.2 Signature of the image

Rotation invariance is achieved by using the binary mask to create the signature of the image. First, the modulus of the FRFT of the spectral image is calculated (Fig. 6a) using the same $\alpha$ angle used to obtain the mask (Fig. 6b), and then the modulus is filtered by the mask (Fig. 6c) as

$$H_{RN} = M_{RN} \cdot \left| \mathcal{F}_\alpha \left\{ I_x \right\} \right|. $$

(10)

After that, the rings in $H_{RN}$ are numbered from the center toward the outside. The final step is to calculate the sum of the values sampled by each ring, to generate a normalized signature $S_{RN}$ of the image (Fig. 6d). In Fig. 7, the signatures for the same image obtained at different values of $\alpha$ are shown.
Figure 6. Procedure to obtain the normalized signature of the image by methodology 1. (a) The modulus of the FRFT of the spectral image using $\alpha = 85^\circ$. (b) Binary rings mask $M_{RN}$. (c) The result of a point to point multiplication of the modulus of the FRFT of the spectral image with $M_{RN}$. (d) Normalized signature for diatom species *Actinocyclus inges Rattray* using method 1.

Figure 7. Normalized signatures of diatom species *Actinocyclus inges Rattray* using method 1 for different $\alpha$. (a) $\alpha = 85^\circ$. (b) $\alpha = 45^\circ$.

2.2 Methodology 2. Working with the FRFT of the image to create the binary rings mask

2.2.1 The $M_{RN}$ binary rings mask

In this methodology, the FRFT is applied to the image to generate the binary mask. The real part of the FRFT of the picture is filtered by the disk $D$ and an optimum profile selected (Fig.8). Next, from the slopes of the optimum profile, a binary function is constructed by using the condition in Eq. (9). Rotation of the binary function generates the binary ring mask $M_{RN}$. Figure 9 shows the binary masks obtained at different $\alpha$ angles of the FRFT for the image of diatom species *Actinocyclus inges Rattray*.

2.2.2 Signature of the image
In this methodology, the modulus of the FT of the image is used to construct the signature. Thus the binary rings mask, is used to filter the modulus of the FT to calculate the normalized signature. This procedure is shown in Fig. 10. In Fig. 11, the signatures for different values of \( \alpha \) are shown.

![Figure 8](image.png)

Figure 8. (a) Image of diatom species *Actinocyclus inges Rattray*. (b) The real part of the FRFT of the image using \( \alpha = 85^\circ \). (c) Optimum profile extracted from the real part of the FRFT.

![Figure 9](image.png)

Figure 9. \( M_{RN} \) mask for diatom species *Actinocyclus inges Rattray* using method 2 for different \( \alpha \). (a) \( \alpha = 85^\circ \). (b) \( \alpha = 45^\circ \).

3. CLASSIFICATION

In the recognition step, nonlinear correlation is used to classify between the signatures of the problem image (PI) and the target image (TI). If \( S_{PI} \) is the signature of a problem image and \( S_{TI} \) is a signature from the target image, the nonlinear correlation or \( k \)-law correlation is defined as

\[
C_{NL} = \mathcal{F}^{-1} \left[ \left| \mathcal{F}\{S_{PI}\} \right|^k \exp\left(i\phi_{S_{PI}}\right) \mathcal{F}\{S_{TI}\} \right|^k \exp\left(-i\phi_{S_{TI}}\right) \right],
\]

where \( k = 0.01 \).
Figure 10. Procedure to obtain the normalized signature of the image by methodology 2. (a) The modulus of the FT of the image. (b) Binary rings mask $M_{RN}$. (c) The result of a point to point multiplication of the modulus of the FT of the spectral with $M_{RN}$. (d) Normalized signature for diatom species *Actinocyclus inges Rattray* using method 2.

Figure 11. Normalized signatures of diatom species *Actinocyclus inges Rattray* using method 2 for different $\alpha$. (a) $\alpha = 85^\circ$. (b) $\alpha = 45^\circ$.

4. **OPTIMUM ANGLE OF THE FRFT**

As seen in Fig. 7 and Fig. 11, changing the $\alpha$ angle of the FRFT changes the signature produced by the binary rings mask. Therefore, the resulting value of the signature correlation is different. Thus, for each methodology, a search for the $\alpha$ angle that gives the highest correlation value must be performed. That angle is called the optimum angle.

To search for the optimum angle, the signature it is obtained for a given $\alpha$. Then the signature is autocorrelated using $k$-law correlation; the process is repeated changing $\alpha$ from $1-90^\circ$ with steps of one degree. The $\alpha$ angle that gives the highest correlation value is the optimum angle for the image and methodology. Figure 12 shows the non-normalized autocorrelation values for each $\alpha$ for methodology 1 (Fig. 12a) and methodology 2 (Fig. 12b). It can be seen that the optimum angle for method 1 is at $\alpha = 60^\circ$ and for method 2 is at $\alpha = 68^\circ$ for *Actinocyclus inges Rattray*.
Figure 12. Maximum autocorrelation values for $\alpha$ angles from 1 to 90 degrees for diatom species *Actinocyclus inges* Rattray.
(a) Methodology 1. (b) Methodology 2.

5. NOISE ANALYSIS

For both methodologies, a noise analysis was performed using the discrimination coefficient metric defined as:

$$ DC = 1 - \frac{\max |C_{NL}(S_{TI}, S_{TN})|^2}{(C(0))^2}, $$

(12)

where $C = |C_{NL}(S_{TI}, S_{TN})|$, $S_{TI}$ is the signature of the target image, $S_{TN}$ is the signature of the target image with noise, $S_{BN}$ is the signature of the background image with noise and $C_{NL}$ is the nonlinear correlation. Image of diatom species *Actinocyclus inges* Rattray was first affected with Gaussian noise of media zero changing variance from 0 to 5 and the $DC$ is calculated, repeating the process 49 times. The same procedure was performed for salt and pepper noise, changing density from 0 to 1. Then, for both types of noise, the mean $DC$ is obtained and ±1.96 standard errors calculated which gives a 95% confidence level.

Figure 13 shows the results for Gaussian noise and Fig. 14 for salt and pepper. Results showed that methodology 2 has better performance for Gaussian noise. For salt and pepper, both methods have almost the same performance, but method 2 has slightly better performance.
6. CONCLUSIONS

This work shows that for both methodologies presented, higher correlation values were achieved by working outside the Fourier plane. Also, the two methods were tested using the same image of diatom species *Actinocyclus inges* Rattray, affected by Gaussian and salt and pepper noise. The results show a better performance of method 2 for both types of noise.
REFERENCES


