Pattern recognition with an adaptive joint transform correlator

Victor H. Diaz-Ramirez, Vitaly Kober, and Josue Alvarez-Borrego

An adaptive joint transform correlator for real-time pattern recognition is presented. A reference image for the correlator is generated with a new iterative algorithm. The training algorithm is based on synthetic discriminant functions. The obtained reference image contains the information needed to reliably discriminate a target against known false objects and a cluttered background. Calibration lookup tables of all optoelectronics elements used are included in the design of the adaptive joint transform correlator. Two methods for the implementation of the proposed joint transform correlator in an opto-digital setup are considered. Experimental results are provided and compared with those of computer simulations. © 2006 Optical Society of America

OCIS codes: 070.4550, 070.5010.

1. Introduction

The design of filters for pattern recognition based on correlation has attracted considerable attention in the past few decades. This is because correlation-based filters can be implemented optically or by using hybrid (opto-digital) systems exploiting the parallelism inherent in optical systems. Consequently these systems are able to carry out the recognition process at a high rate. Note that hybrid systems with the use of liquid crystal displays (LCDs) are flexible. The electrically addressed nature of a LCD utilized as a spatial light modulator (SLM) permits flexible programming and data manipulation with a computer. Hybrid systems can be implemented on the basis of two principal architectures, that is, the 4f correlator (4FC) and the joint transform correlator (JTC). The advantage of the JTC compared to the 4FC is that the former is less sensitive to misalignments of an optical setup such as scale, horizontal, vertical, and azimuthal differences between the input and frequency planes. The common way to design correlation filters is to produce filters that optimize different criteria. Many performance measures for correlation filters have been proposed and summarized. Some of these measures can essentially be improved using an adaptive approach to the filter design. According to this concept, we are interested in a filter with good performance characteristics for a given observed scene, i.e., with a fixed set of patterns or a fixed background to be rejected, rather than in a filter with average performance parameters over an ensemble of images. The response of correlation filters also depends on the scale, orientation, and any deformation in the input object. One of the solutions of the problem is to take into account the information about objects to be recognized and objects to be rejected by means of synthetic discriminant functions (SDFs). SDF filters are an attractive technique for distortion-invariant pattern recognition. Originally this technique was proposed on the basis of the 4FC. Many efforts were made to find an effective implementation of SDFs with the JTC. The most successful schemes that improve the performance of conventional JTCs are the nonlinear joint transform correlator and the fringe-adjusted joint transform correlator. These techniques significantly improve the discrimination capability of pattern recognition when a target is added or embedded into a background noise. However, these techniques often fail to recognize a desired object when the shape of the undesired objects is similar to that of a target. Moreover, these methods are still sensitive to the small geometric distortions of the objects to be recognized. Recently iterative algorithms for the design of
adaptive 4FC were suggested. In this work we propose a new iterative method for the design of an adaptive JTC. The purpose of the design is the improvement of the recognition process with the JTC by exploiting the adaptive approach that is directly connected to the correlator’s discrimination capability. By training of the conventional SDF filter, the algorithm generates an adaptive robust SDF filter that can serve as a reference image in the JTC. The proposed method is able to take into account calibration lookup tables of all optoelectronic devices used in real experiments. The obtained reference image contains the information about an object and its variants to be recognized and undesired objects including a background to be rejected. The designed adaptive JTC is able to discriminate at a high rate a geometrically distorted version of a target against known false objects embedded into a cluttered background. This paper is organized as follows. In Section 2 we present a brief review of the classical JTC, the nonlinear JTC, and the fringe-adjusted JTC. A short review of SDF filters and the proposed iterative method is described in Section 3. Two methods for optodigital implementation of the proposed adaptive JTC are considered in Section 4. In Section 5 computer simulation and experimental results are provided and discussed. Section 6 summarizes our conclusions.

2. Joint Transform Correlators

The JTC was originally introduced by Weaver and Goodman in 1966. The main drawback of the classical JTC is its sensitivity to geometric distortions and noise in the input scene. Several partial solutions have been proposed for this architecture to improve the correlator performance with respect to different measures. Two of these variants are the nonlinear JTC and the fringe-adjusted JTC. In the former a nonlinear elementwise transformation of the joint power spectrum before the inverse Fourier transform is carried out. In the latter a real-valued filter in the frequency domain is applied to the joint power spectrum before the inverse Fourier transform. It has been shown that these two JTCs yield better performance compared to that of the classical JTC in terms of correlation peak intensity, correlation width, and discrimination capability.

A. Classical Joint Transform Correlator

A block diagram of the classical JTC is shown in Fig. 1. The input plane (joint image) \( f(x, y) \) consists of a scene image \( s(x, y) \) alongside a reference image \( t(x, y) \), which are separated by a distance, say \( 2d \). The scene image can contain objects (desired and undesired) embedded in a background. The joint image can be written as

\[
f(x, y) = s(x, y - d) + t(x, y + d),
\]

with its Fourier transform:

\[
F(\mu, \nu) = S(\mu, \nu) \exp(-i\nu) + T(\mu, \nu) \exp(i\nu).
\]

The joint power spectrum obtained (for instance, with a CCD camera) is given by

\[
E(\mu, \nu) = |F(\mu, \nu)|^2 = S(\mu, \nu)S^*(\mu, \nu) + T(\mu, \nu)T^*(\mu, \nu) + S(\mu, \nu)T^*(\mu, \nu)\exp(-2i\nu d) + S^*(\mu, \nu)T(\mu, \nu)\exp(2i\nu d).
\]

Applying the inverse Fourier transform to Eq. (3) we obtain

\[
e(x, y) = s(x, y) \otimes s(x, y) + t(x, y) \otimes t(x, y) + s(x, y - 2d) \otimes t(x, y - 2d) + s(x, y + 2d) \otimes t(x, y + 2d),
\]

where \( \otimes \) denotes the correlation operation. It can be seen from Eq. (4) that the autocorrelations of the scene and target images mainly contribute at the origin, whereas the cross-correlation terms, which are terms of interest, are placed at the distances \( \pm 2d \). A drawback of the classical JTC is its extremely low tolerance to noise when objects are embedded in a nonstationary background noise. The performance of the classical JTC with respect to the discrimination capability (DC) is poor. The DC is formally defined as the ability of a filter to distinguish a target among other different objects. If a target is embedded in a background that contains false objects, then the DC can be expressed as follows:

\[
DC = 1 - \frac{|C^B(0, 0)|^2}{|C^T(0, 0)|^2},
\]

where \( |C^B|^2 \) is the maximum intensity in the correlation plane over the area of the background to be rejected and \( |C^T|^2 \) is the maximum intensity in the correlation plane over the area of the target position. The area of the target position is determined in close vicinity to the actual target location. The area of the background is complementary to the area of the target position. Further accurate estimation of the target location with correlation filters can be carried out. Negative values of the DC indicate that a tested filter fails to recognize the target. Assume that an input image \( f(x, y) \) contains the input objects \( s(x, y) \) (desired and undesired) and the nonoverlapping background \( b(x, y) \):
The joint power spectrum is given by

\[ P(f, k) = |F(f, k)|^2 = S(f, k) + T(f, k) + \hat{B}(f, k) \]

where

\[ \hat{B}(f, k) = \hat{B}(x, y) = w(x - x_0, y - y_0) \hat{b}(x, y), \]

and \((x_0, y_0)\) are unknown coordinates of the target in the input scene; \(w(x - x_0, y - y_0)\) is a binary function defined as

\[ w(x - x_0, y - y_0) = \begin{cases} 0, & \text{within the object area} \\ 1, & \text{otherwise} \end{cases} \]

The joint power spectrum is given by

\[
|F(f, k)|^2 = S(f, k) + T(f, k) + \hat{B}(f, k)
\]

\[
+ \left[ T(f, k)S^*(f, k) + T^*(f, k)\hat{B}^*(f, k) \right] \exp(-i2\pi f d)
\]

\[
+ \left[ T^*(f, k)S(f, k) + T(f, k)\hat{B}^*(f, k) \right] \exp(i2\pi f d).
\]  

Note that the joint power spectrum contains the Fourier transforms with the phase factors of \(\exp(\pm i2\pi f d)\) corresponding to the cross-correlation terms between the target and the input objects, the target and the background, and the input objects and the background. The latter correlation term severely affects the DC.

B. Nonlinear Joint Transform Correlator

The nonlinear JTC performs a pointwise, nonlinear transformation of the joint power spectrum. A block diagram of this correlator is shown in Fig. 2. The nonlinearly transformed joint power spectrum can be considered as the output of a \(k\)th-law nonlinear system.\(^{11,13}\) Let \(E\) be the joint power spectrum given in Eq. (3), and let \(g(E)\) be a nonlinear function\(^{13}\):

\[ g(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \exp(i\omega E) d\omega. \]  \hspace{1cm} (11)

The nonlinearly transformed joint power spectrum can be obtained by substituting Eq. (3) into Eq. (11):

\[ g(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) \exp[i\omega(S^2(\mu, \nu) + T^2(\mu, \nu))]\]

\[ \times \exp[i2\omega S(\mu, \nu)T(\mu, \nu) \]

\[ \times \cos[2\nu\phi_s(\mu, \nu) - \phi_T(\mu, \nu)] d\omega, \]

where \(S(\mu, \nu)\) and \(T(\mu, \nu)\) are the Fourier transforms of \(s(x, y)\) and \(t(x, y)\), respectively, and \(\phi_s(\mu, \nu)\) and \(\phi_T(\mu, \nu)\) are the phases of \(S(\mu, \nu)\) and \(T(\mu, \nu)\), respectively. Assume that the Fourier transform of the odd full-wave \(k\)th-law device is given by\(^{11}\)

\[ G(\omega) = \frac{2}{(i\omega)^{k+1}} \Gamma(k + 1), \]  \hspace{1cm} (13)

where \(\Gamma\) is the gamma function and \(k\) is the severity of the nonlinearity \((k = 1\) corresponds to a linear device, \(k = 0\) corresponds to a hard clipping nonlinearity). Applying the Jacobian–Anger formula to Eq. (12), the output of the nonlinear system can be written as follows\(^{14}\):

\[ g_k(E) = \sum_{v=0}^{\infty} \epsilon_v \gamma(k + 1)[S(\mu, \nu)T(\mu, \nu)]^k \]

\[ \times \cos[2\nu\phi_s(\mu, \nu) - \phi_T(\mu, \nu)], \]  \hspace{1cm} (14)

where

\[ \epsilon_v = \begin{cases} 1, & \text{for } v = 0 \\ 2, & \text{for } v > 0. \end{cases} \]

It can be seen from Eq. (14) that each harmonic term is phase modulated by \(v\) times the phase difference of the input signal and the reference signal Fourier transforms, and the higher-order correlation signals are diffracted to \(2v\)d. By varying \(k\), correlation signals with different characteristics can be produced. For example, for \(k = 1\) the system is equal to the classical JTC, for \(k = 2\) the system has the phase-only response, and for \(k = 0\) the system is identical to the binary JTC.\(^{12}\)

C. Fringe-Adjusted Joint Transform Correlator

A block diagram of the fringe-adjusted JTC is shown in Fig. 3. In this scheme the joint power spectrum is multiplied by the frequency response of a fringe-adjusted filter before applying the inverse Fourier transform. The frequency response of the fringe-adjusted filter is defined by\(^{15}\)
where \( B(\mu, v) \) is a function to control the gain and \( A(\mu, v) \) is a function to avoid pole problems. When \( B(\mu, v) = 1 \) and \( |T(\mu, v)|^2 \gg A(\mu, v) \), then the frequency response of the filter approaches

\[
H_{\text{frf}}(\mu, v) \approx \frac{1}{|T(\mu, v)|^2}. \tag{16}
\]

So if the reference and target images are identical, then the fringe-adjusted joint power spectrum can be written as

\[
G(\mu, v) = 2\{1 + \cos[\phi_s(\mu, v) - \phi_T(\mu, v) + 2\pi d]\}. \tag{17}
\]

This means that the correlation plane contains two peaks located at \( \pm 2d \) and a peak at the origin.

3. Filter Design

A. Synthetic Discriminant Function Filter Background

Basically, SDF filters can be used for multiclass distortion-invariant pattern recognition. Let us define the true class as different versions of an object to be recognized. The false class consists of all undesired objects and a background to be rejected. In this case, a correlation filter can be constructed as a linear combination of classical matched filters (CMFs)\(^1\) for different objects. The weighting coefficients in the linear combination are determined in such a manner that the cross correlations at the origin should be equal for all objects belonging to the same class. Let \( \{s_i(x, y), i = 1, 2, \ldots, N\} \) be a set of (linearly independent) training images each with \( M \) pixels. The SDF filter function \( h(x, y) \) in the space domain can be expressed as a linear combination of the set of the training images, i.e.,

\[
h(x, y) = \sum_{i=1}^{N} a_i s_i(x, y), \tag{18}
\]

where \( \{a_i, i = 1, 2, \ldots, N\} \) are the weighting coefficients, and they are chosen to satisfy the following conditions:

\[
s_i \cdot h = c_i. \tag{19}
\]

Here the symbol \( \cdot \) represents the inner product, and \( \{c_i, i = 1, 2, \ldots, N\} \) are prespecified values in the correlation output at the origin for each training image. Let \( R \) denote a matrix with \( N \) columns and \( M \) rows (the number of pixels in each training image), where its \( i \)th column is given by the vector version of \( s_i(x, y) \). Let \( a \) and \( c \) represent column vectors of \( a_i \) and \( c_i \), respectively. We can rewrite Eqs. (18) and (19) in matrix-vector notation as follows:

\[
h = Ra, \tag{20}
\]

\[
c = R^T h, \tag{21}
\]

where the superscript + means the conjugate transpose. The \((i, j)\)th element of the matrix \( S = (R + R^T) \) is the value at the origin of the cross correlation between the training images \( s_i(x, y) \) and \( s_j(x, y) \). If the matrix \( S \) is nonsingular, the solution of the equation system is given by

\[
a = (R^T R)^{-1} c, \tag{22}
\]

and the filter vector is

\[
h = R(R^T R)^{-1} c. \tag{23}
\]

The SDF filter with equal output correlation peaks can be used for intraclass distortion-invariant pattern recognition, i.e., detection of distorted patterns belonging to the true class of objects. This can be done by setting all elements of \( c \) to unity, i.e.,

\[
c = [1, 1, \ldots, 1]^T. \tag{24}
\]

Now we consider a two-class recognition problem. Suppose that there are \( N \) training images from a true class and \( K \) training images from a false class. To recognize training images from the true class and to reject training images from the false class, we set the filter output \( \{c_i = 1, i = 1, 2, \ldots, N\} \) for the true class objects and \( \{c_i = 0, i = N + 1, N + 2, \ldots, N + K\} \) for the false class objects, i.e.,

\[
\text{Fig. 3. Block diagram of the fringe-adjusted JTC.}
\]

\[
H_{\text{frf}}(\mu, v) = \frac{B(\mu, v)}{A(\mu, v) + |T(\mu, v)|^2}, \tag{15}
\]

\[
\text{Fig. 4. Block diagram of the proposed iterative algorithm.}
\]
Using the filter given in Eq. (23) for pattern recognition, we expect that the central correlation peak will be close to unity for the true class objects, and it will be close to zero for the false class objects. Obviously the above approach can be easily extended to any number of classes to be discriminated. Note that this simple procedure lacks control over the full correlation output because we are able to control only the correlation output at the locations of the cross-correlation peaks. So other sidelobes (false peaks) may appear everywhere on the correlation plane.

B. Design of Adaptive Joint Transform Correlators

Now we state the pattern recognition problem to be solved. We wish to design a JTC that ensures a high correlation peak corresponding to the target while suppressing possible false peaks. In other words, to achieve good recognition of the target, it is necessary to reduce correlation function levels at all false peaks except at the origin of the correlation plane, where the constraint on the peak value must be met. For a given object to be recognized and for false objects and a background to be rejected, an iterative algorithm is used. At each iteration the algorithm suppresses the highest sidelobe peak and therefore monotonically increases the value of discrimination capability until a prespecified value is reached. We are interested in a correlation filter that identifies the target with a high discrimination capability in cluttered and noisy input scenes. Actually, in this case conventional cor-

\[ \mathbf{c} = [1, 1, \ldots, 1, 0, 0, \ldots, 0]^T. \]  

(25)

Using the filter given in Eq. (23) for pattern recognition, we expect that the central correlation peak will be close to unity for the true class objects, and it will be close to zero for the false class objects. Obviously the above approach can be easily extended to any number of classes to be discriminated. Note that this simple procedure lacks control over the full correlation output because we are able to control only the correlation output at the locations of the cross-correlation peaks. So other sidelobes (false peaks) may appear everywhere on the correlation plane.

B. Design of Adaptive Joint Transform Correlators

Now we state the pattern recognition problem to be solved. We wish to design a JTC that ensures a high correlation peak corresponding to the target while suppressing possible false peaks. In other words, to achieve good recognition of the target, it is necessary to reduce correlation function levels at all false peaks except at the origin of the correlation plane, where the constraint on the peak value must be met. For a given object to be recognized and for false objects and a background to be rejected, an iterative algorithm is used. At each iteration the algorithm suppresses the highest sidelobe peak and therefore monotonically increases the value of discrimination capability until a prespecified value is reached. We are interested in a correlation filter that identifies the target with a high discrimination capability in cluttered and noisy input scenes. Actually, in this case conventional cor-

![Diagram of a compact JTC used in experiments.](image1)

**Fig. 5.** (Color online) Compact JTC used in experiments.

![Intensity response of a twisted nematic LCD captured with a CCD camera.](image2)

**Fig. 6.** Intensity response of a twisted nematic LCD captured with a CCD camera.

![Input scene containing two objects with the same shape but with different information contents.](image3)

**Fig. 7.** Input scene containing two objects with the same shape but with different information contents.
relation filters may yield poor performance. With the help of adaptive SDF filters, a given value of the DC can be achieved. The algorithm of the filter design requires knowledge of the background image. The background can be described stochastically or deterministically. The first step is to carry out the joint transform correlation between the background (deterministic picture or realization of stochastic process) and a basic SDF filter, which is initially trained only with the target. Next the intensity maximum of the filter output is set as the origin, and around the origin we form a new object to be rejected from the background. The created object is added to the false class of objects. Now a two-class recognition problem is utilized to design a new SDF filter; that is, the true class contains only the target and the false class consists of the false-class objects. The described iterative procedure is repeated until a given value of the DC is obtained. Finally, note that if other objects to be rejected are known, they can be directly included in the false class and used for the design of the adaptive SDF filter. A block diagram of the procedure is shown in Fig. 4. So the proposed algorithm consists of the following steps:

(i) Create a basic SDF filter trained only with the target using Eq. (23).
(ii) Create the input image [see Eq. (1)] by composing the designed SDF filter and the image to be rejected (nondesired objects or a background).
(iii) Carry out the joint transform correlation including calibration lookup tables of all optoelectronics devices such as a real SLM and a CCD camera.
(iv) Calculate the DC using Eq. (5).
(v) If the value of the DC is greater or equal to the

Fig. 8. Computer simulation results obtained for the scene in Fig. 7 with (a) binary JTC, (b) fringe-adjusted JTC.

Fig. 9. Bipolar reference image obtained for the scene in Fig. 7 with the proposed method.

Fig. 10. Computer simulation result obtained for the scene in Fig. 7 with the proposed JTC.

Fig. 11. Input scene containing three objects with similar shapes but with different information contents.
desired value, then the filter design procedure is finished; otherwise, go to the next step.

(vi) Create a new object to be rejected from the background. The origin of the object is at the highest sidelobe position in the intensity correlation plane. The region of support of the new object is the union of the shapes of all objects involved in the process (desired and nondesired objects). The object is included in the false class of objects.

(vii) Design a new SDF filter utilizing the two-class recognition problem. The true class contains only the target and the false class consists of the false-class objects. Go to step (ii).

4. Optodigital Implementation

Twisted nematic LCDs are widely used for real-time pattern recognition. Under the proper conditions of applied voltage and input polarizations, these kinds of modulators can produce amplitude-only and phase-only modulations. Their important characteristics are as follows:

(i) They are electrically controlled with conventional video signals.

(ii) They can operate as amplitude-only or phase-only modulators by changing the direction of the polarization vector of the incident light.

(iii) They are able to operate at the speed of conventional television standards.

(iv) These devices can handle a dynamic range of $[0–255]$ for amplitude modulation and a phase range of $[0–2\pi]$ for phase modulation.

In general, the impulse response of SDF filters is a bipolar image. To introduce these kinds of images into spatial light modulators we use two methods described in Subsections 4.A and 4.B.

A. Bipolar Decomposition Method

This method consists of the decomposition of a bipolar image into two images with nonnegative values. In other words, decomposed images are positive and
negative parts of the original image. Next, two JTC operations with nonnegative input images are independently carried out and followed by pointwise operations over the obtained correlation planes form the output of the JTC. A bipolar image can be expressed as

\[ h(x, y) = h^+(x, y) - h^-(x, y), \]  
(26)

where

\[
\begin{align*}
  h^+(x, y) &= \begin{cases} 
    h(x, y), & h(x, y) \geq 0 \\
    0, & \text{otherwise}
  \end{cases}, \\
  h^-(x, y) &= \begin{cases} 
    h(x, y), & h(x, y) < 0 \\
    0, & \text{otherwise}
  \end{cases}.
\end{align*}
\]
(27)

(28)

Now assume that in the model of the joint image [see Eq. (1)] the distance \( d \) is large. So in the output of the JTC [see Eq. (4)], we can ignore the influence of autocorrelation terms to cross-correlation areas. In this case the intensity of cross correlation between \( s(x, y) \) and \( h(x, y) \) may be written as

\[
\begin{align*}
  c(x, y) &= |s(x, y) \otimes [h^+(x, y) - h^-(x, y)]|^2 \\
  &= |s(x, y) \otimes h^+(x, y)|^2 + |s(x, y) \otimes h^-(x, y)|^2 \\
  &\quad - 2 |s(x, y) \otimes h^+(x, y) \cdot h^-(x, y)|^{1/2} \\
  &\quad \times \left[ |s(x, y) \otimes h^+(x, y)|^2 \right]^{1/2}.
\end{align*}
\]
(29)

It can be seen from Eq. (29), with the help of decomposition and simple postprocessing, how to obtain the output of the JTC when the reference image has positive and negative values.

B. Constant Addition Method

The idea of the method is to transform the input-composed bipolar image into an input-composed nonnegative image. It can be easily done by adding a bias value to the input bipolar image. Next the JTC operation with the input-composed nonnegative image is...
where the joint image can be written as one optical correlation. The transformed nonnegative part of the reference image in Fig. 9.

![Fig. 16. Joint images composed with the scene in Fig. 7 and with (a) the positive part of the reference image in Fig. 9 and (b) the negative part of the reference image in Fig. 9.](image)

performed. Simple postprocessing is required to obtain the output of the JTC. Note that we need only one optical correlation. The transformed nonnegative joint image can be written as

\[
f(x, y) = \tilde{s}(x, y - d) + \tilde{h}(x, y + d),
\]

where \(\tilde{s}(x, y) = s(x, y) + c\) and \(\tilde{h}(x, y) = h(x, y) + c\), \(s(x, y)\) are the scene images, \(h(x, y)\) is the bipolar reference image, and \(c = \min[h(x, y)]\) is a constant value. The intensity output of the JTC with the new joint image is given by [see Eq. (4)]

\[
c(x, y) = |\tilde{s}(x, y) \otimes \tilde{s}(x, y) + \tilde{h}(x, y) \otimes \tilde{h}(x, y)|^2
+ |\tilde{s}(x, y - 2d) \otimes \tilde{h}(x, y - 2d)|^2
+ |\tilde{h}(x, y + 2d) \otimes \tilde{s}(x, y + 2d)|^2.
\]

The two latter terms of Eq. (31) are the terms of interest. In this case we also suppose that the influence of autocorrelation terms to cross-correlation areas is low. The intensity of the cross correlation between \(s(x, y)\) and \(h(x, y)\) can be computed from the intensity of the cross correlation between nonnegative images as follows:

\[
|s(x, y) \otimes h(x, y)|^2 = \left|\left[s(x, y) + c - c\right] \otimes \left[h(x, y) + c - c\right]\right|^2
= \left|\left[\tilde{s}(x, y) - c\right] \otimes \left[\tilde{h}(x, y) - c\right]\right|^2
= \left|\tilde{s}(x, y) \otimes \tilde{h}(x, y)\right|^2
+ \left|\tilde{h}(x, y) \otimes c\right|^2
+ \left|\tilde{s}(x, y) \otimes c\right|^2
+ \left|c \otimes c\right|^2
- 2\left[\tilde{s}(x, y) \otimes \tilde{h}(x, y) \cdot \tilde{h}(x, y) \otimes c\right]
- 2\left[\tilde{s}(x, y) \otimes \tilde{h}(x, y) \cdot \tilde{s}(x, y) \otimes c\right]
+ 2\left[\tilde{h}(x, y) \otimes c \cdot \tilde{s}(x, y) \otimes c\right]
+ 2\left[\tilde{s}(x, y) \otimes \tilde{h}(x, y) \cdot c \otimes c\right]
- 2\left[\tilde{h}(x, y) \otimes c \cdot c \otimes c\right]
- 2\left[\tilde{s}(x, y) \otimes c \cdot c \otimes c\right].
\]

Further simplifying, we can rewrite Eq. (32) as

![Fig. 17. Cross-correlation intensity planes obtained in optodigital JTC for the joint images shown in (a) Figs. 16(a) and (b) 16(b).](image)
Here $\hat{s}(x, y) \otimes \tilde{h}(x, y)$ can be obtained by applying the pointwise square root to the intensity $|s(x, y) \otimes h(x, y)|^2$.

Fig. 18. Cross-correlation intensity plane after elementwise postprocessing: (a) intensity plane, (b) intensity distribution.

\[
|s(x, y) \otimes h(x, y)|^2 = \left| \hat{s}(x, y) \otimes \tilde{h}(x, y) \right|^2 + C_1^2 + C_2^2 + C_3^2 - 2\{\hat{s}(x, y) \otimes \tilde{h}(x, y) \cdot C_1\}
\]

Fig. 19. Joint image formed for the constant addition method.
\[ h(x, y) \], and constants \( C_1 = h(x, y) \otimes c, \]
\[ C_2 = s(x, y) \otimes c, \]
\[ C_3 = c \otimes c \]
are computed in the following way:
\[
C_1 = \alpha \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x + \tau_x, y + \tau_y) d\tau_x d\tau_y \right]
\]
\[
C_2 = \alpha \left[ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} s(x, y) d\tau_x d\tau_y \right]
\]
\[
C_3 = \alpha c^2
\]
where \( \alpha \) is a normalization factor and the symbol \( \sum [\cdot] \) denotes the summation of all elements of the image.

5. Results

A. Calibration of Devices

Real experiments were carried out using a compact JTC architecture, which is shown in Fig. 5. First we characterized optoelectronics devices such as a twisted nematic LCD of 800 \( \times \) 600 pixels and a monochrome CCD camera of 640 \( \times \) 480 pixels. The twisted nematic LCD worked in the amplitude-only modulation regime. The experimental calibration lookup table of the intensity response of the LCD captured with the CCD camera is shown in Fig. 6. It can be seen that a gray-scale dynamic range is [0–48]. It is interesting to note that in this range the plot is nonlinear due to quantization effects, and it is well approximated with \( k \)-th law nonlinearity
\[
Output = \left( \left| Input \right|^k \right)^k \text{ when } k = 0.7.
\]
We used this information in the iterative process of the adaptive JTC design. This means that input images were transformed from the range [0–255] to the range [0–48] and then used in the iterative algorithm for the design of the nonlinear JTC with a nonlinearity degree of \( k = 0.7 \).

B. Computer Simulation Results

The size of all monochrome images used in our experiments is 128 \( \times \) 128 pixels. The signal range is [0–255]. Figure 7 is an example of the input scene. The scene contains two objects with similar shape and size (approximately 44 \( \times \) 28 pixels) but with different gray-level contents. The target is the upper butterfly with black wings. The objects are embedded into an aerial picture at unknown coordinates. We compare the performance of the proposed adaptive JTC with those of the binary JTC and the fringe-adjusted JTC. The correlation intensity planes obtained with the latter two systems are shown in Fig. 8. We see that these two systems fail to discriminate the target against the false object with a similar shape. The performance of the adaptive JTC in the design process after eight iterations reaches DC = 0.95. The obtained bipolar reference image is shown in Fig. 9. Next we test the recognition performance with the adaptive JTC when the objects are placed into the background at arbitrary coordinates. We used 50 statistical trials of our experiment for different positions of the target. With 95% confidence, the DC is equal to 0.82 \( \pm \) 0.003. The correlation intensity plane obtained with the adaptive JTC for the input scene in Fig. 7 is shown in Fig. 10. Now we investigate the tolerance of the adaptive JTC to small geometric image distortions. Several methods have been proposed to improve the pattern recognition in the presence of such distortions. These methods can be broadly classified into two groups. The first class is formally concerned with two-dimensional scaling and rotation distortions. Such methods include space-variant transforms and circular harmonic functions. The second class of filters uses training images that are sufficiently descriptive and representative of the expected distortions. The proposed method is based on the second approach. In our experiments the input scene shown in Fig. 11 with an embedded geometrically distorted target and two false objects with similar shapes is used. The target is the upper-right butterfly. Fifty statistical trials of each experiment for different positions of a distorted target are carried out. First, geometric distortion by means of rotation is investigated. The adaptive JTC is trained with two versions of the object rotated by 0° and 5°. After 41 iterations the filter yields DC = 0.95. The obtained reference image is shown in Fig. 12. Next we test the performance of the adaptive JTC with the input scene in Fig. 11 when the target is rotated by 0°, 2°, 4°, 6°, 8°, and 10°. The results in terms of the correlation intensity distributions are shown in Fig. 13. It can be
seen that the proposed filter adapts well by training with small rotations of the target. Next tolerance of the filter to scale distortions of the target is investigated. The adaptive JTC is trained with three versions of the object scaled by factors of 0.95, 1, and 1.15. After 35 iterations the obtained filter yields DC = 0.95. The obtained reference image is shown in Fig. 14. Next we test the performance of the adaptive JTC with the input scene in Fig. 11 when the target is scaled by factors of 0.98, 0.99, 1.0, 1.1, and 1.2. The results in terms of the correlation intensity distributions are shown in Fig. 15. One can observe that the filter always detects the scaled object. In a similar manner, the proposed method can be used to design an adaptive JTC that can have good tolerance to the arbitrary geometric distortions. The complexity of the composite filter design depends on the size of the training set of distorted images used.

C. Experimental Results

In this section we present the experimental results obtained with the two methods of JTC implementation described in Section 4. The input scene and the computed reference image shown in Figs. 7 and 9, respectively, are used in the bipolar decomposition method experiment. The input scene and the computed reference image shown in Figs. 11 and 12, respectively, are used to test the constant addition method. The experimental results are compared with those of computer simulations.

1. Bipolar Decomposition Method Results

The first optodigital experiment is based on the bipolar decomposition method. The reference image in Fig. 9 has real positive and negative values. So we decompose this image into two nonnegative images (see Section 4). Two experiments are performed. In the first experiment the input scene is composed with the positive part of the reference image and the joint transform correlation is carried out. The experiment is repeated with the negative part of the reference image. The composed images are shown in Fig. 16. The intensity correlation outputs of JTC are shown in Fig. 17. The correlation plane obtained after the post-processing given in Eq. (29) is shown in Fig. 18. The DC obtained in the experiment is equal to 0.78.

2. Constant Addition Method Results

The second optodigital experiment is based on the constant addition method described in Section 4. The joint input image used in the optodigital JTC is shown in Fig. 19. The SLM has a finite size (less than the size of the optical lens), and, after adding a high constant bias to the joint image, the signal at the plane of the SLM may be considered as a signal masked by a rectangular window. The Fourier transform of such a signal is the convolution between the spectrum of the joint image and a sinc function (Fourier transform of the rectangular window). Actually, the sinc function has high sidelobes that may severely affect the joint power spectrum. To avoid these effects, the input joint image is masked by a window with smoothed edges. Next we calculate all needed constants $C_1, C_2,$ and $C_3$ given in Eq. (34). Figure 6 gives the relationship between a dynamic range of the used optodigital LCD and CCD camera and a digital range of a signal. Whereas digital images have a range of $[0–255]$ gray-scale levels, the signals in the optodigital domain have a range of $[0–48]$ levels. We need to scale all images and the constant bias involved in the optodigital setup. The needed constants are $C_1 = 31.7546, C_2 = 23.0368,$ and $C_3 = 40,$ and the $\alpha$ value can be estimated as $\alpha = 1/cs,$ where $s$ is the number of image pixels. The cross-correlation intensity plane obtained in the optodigital JTC for the joint image in Fig. 19 is shown in Fig. 20. The cross-correlation intensity plane obtained after the pointwise postprocessing is shown in

![Cross-correlation plane obtained from the plane in Fig. 20 after postprocessing; (a) intensity plane, (b) intensity distribution.](a) (b)
Fig. 21. One can observe that the target is successfully recognized with DC = 0.648. Finally, note that this method requires only one correlation, whereas the bipolar decomposition method uses two correlations to reconstruct the desired output.

6. Conclusions
A new adaptive JTC was proposed to improve the recognition of a target embedded in a known cluttered background. It was shown that the proposed iterative JTC design algorithm with few training iterations helps us take control over the whole correlation plane. The computer simulation results demonstrated superiority in the performance of the proposed JTC for pattern recognition compared to that of the classical JTC, the binary JTC, and the fringe-adjusted JTC. The suggested filter has high scene adaptivity and good robustness to small geometric image distortions. Two methods of optodigital JTC implementation were considered. We have presented computer simulations and experimental results. Very good accordance between these results was obtained.

References