Two-dimensional glitter function in the study of rough surfaces via remote sensing

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Abstract. The van Cittert–Zernike theorem is used to obtain in a rigorous way the bidimensional glitter function used in the problem of retrieving spatial information of surface heights from aerial images. The results show that the glitter function can be described by a circular function where the radius is a function of the angular extent and the position of the source and the viewing direction. This result comes from considering the source as a circular one that is projected on the detector or camera after reflection, taken into consideration the slopes of the surface and the geometrical optics.

1. Introduction

Recently, the problem of retrieving spatial information of sea surface heights from aerial images was considered [1]. As a first step, two kinds of roughness spectra were presented: two roughness spectra which describe single scale surfaces, and the Pierson–Moskowitz power spectrum [2], which describes multiscale surfaces for a fully developed sea. In this first step, surfaces that are rough in one direction were considered. As a second step, a bidimensional surface \( \zeta(x, y) \) was generated for its respective analysis.

The nonlinear technique showed three processes: the surface, surface slopes and the image. For understanding the information contained in the image, the use of a 'glitter function', \( B(M) \) or \( B(M_x, M_y) \) was shown, depending whether one is working in one or two dimensions. This function operates on the slopes of the surfaces. In the particular case of two dimensions, the glitter function was approximated by a Gaussian function [1]. The physical meaning of 'glitter function' can be explained using figure 1. The surface is illuminated by a source, \( \sigma \), of a fixed angular extent, \( \beta \), and imaged through a lens that subtends a very small solid angle. In other words, light from the source is reflected on the surface just one time and, depending on the slopes, the light reflected will or will not be part of the image. In broad terms, the image consists of bright and dark regions that I call 'glitter patterns'. The glitter patterns obtained in the image contain spatial information of the surface. Now, the spatial position and extension of the glitter patterns are calculated with the glitter function.

The glitter function contains information on the angular extent of the source, the incidence angle of the light and the viewing direction. It comes from a projection of the source on the camera or detector after reflection, taking into consideration the slopes of the surface and the geometrical optics.

My aim here is to present in a rigorous way the derivation of a more justifiable bidimensional glitter function, \( B(M_x, M_y) \). This objective is important because...
knowledge of the glitter function is an important step to the application of this nonlinear technique to study surfaces with Gaussian statistics.

2. Bidimensional glitter function

Consider the situation shown in figure 1. The position of the source \( \sigma \), which subtends an angle \( \beta \), is determined by the angles \((\theta, \phi)\), or equivalently by the coordinates \((x_s, y_s, z_s)\). The viewing direction is denoted by the angles \((\theta_d, \phi_d)\), or equivalently by the coordinates \((x_d, y_d, z_d)\). The bidimensional surface is defined by the function \( C(x, y) \), and \( r_o \) and \( r_o' \) denote the distances from the source to the points \( P_1(x_o, y_o) \) and \( P_2(x_o', y_o') \) on the plane \( z=0 \) respectively.

For a quasi-monochromatic source, the mutual intensity \( J_0(x_o, y_o; x_o', y_o') \) on the plane \( z=0 \) may be found using the van Cittert–Zernike theorem

\[
J_0(x_o, y_o; x_o', y_o') = \int \int I(x_0, y_0) \frac{\exp[i\vec{k}(r_o-r_o)']}{r_o r_o'} dx_0 dy_0,
\]

where \( I(x_0, y_0) \) is the intensity per unit area of the source and \( \vec{k} \) is the average wave-number. In the bidimensional case, \( I(x_0, y_0) = \text{Circ}(x_0, y_0) \) (where \( \text{Circ} \) is the circle function and it can be represented like \( \text{rect}[\rho/\beta] \), where

\[
\rho^2 = (x_0-x)^2 + (y_0-y)^2
\]

and \( x_s, y_s \) are the centre of the source). In the Fresnel approximation, the distances \( r_o \) and \( r_o' \) are given by

\[
r_o \approx \frac{1}{2} \frac{(x_0-x)^2}{z_s} + \frac{1}{2} \frac{(y_0-y)^2}{z_s}, \quad (2a)
\]

\[
r_o' \approx \frac{1}{2} \frac{(x_0'-x)^2}{z_s} + \frac{1}{2} \frac{(y_0'-y)^2}{z_s}, \quad (2b)
\]
so that, equation (1) can be written approximately as

\[ J_0(x_0, y_0; x'_0, y'_0) = \exp \left( i \psi \right) \left\{ \text{Circ} \left( x_{0s}, y_{0s} \right) \right\} \times \exp \left\{ -iK \left[ \frac{x_{0s}}{z_s} + \frac{(y_0 - y'_0) y_{0s}}{z_s} \right] \right\} dx_{0s} dy_{0s}, \quad (3) \]

where \( \psi = k [(x_0^2 + y_0^2) - (x'_0^2 + y'_0^2)]/2z_s \).

For a point source at \( \mathbf{R}_1 \cdot \mathbf{n} \), and a viewing direction given by \( \mathbf{R}_2 \cdot \mathbf{n} \), where \( \mathbf{R}_1 \) is a unit vector representing the incident light on the surface, \( \mathbf{R}_2 \) is a vector representing the reflection light to the camera or detector, and \( \mathbf{n} \) is the normal vector to the surface, the phase variations introduced by the surface are approximately given by [3],

\[ \phi(x_0, y_0) = k [\cos (\mathbf{R}_1 \cdot \mathbf{n}) + \cos (\mathbf{R}_2 \cdot \mathbf{n})] \zeta(x_0, y_0). \quad (4) \]

Then, after reflection the mutual intensity on the plane \( z = 0 \), can be written as

\[ J_0(x_0, y_0; x'_0, y'_0) = \int \int \int J_0(x_0, y_0; x', y') \mathcal{K}(x_0, y_0; x, y) \mathcal{K}^*(x, y; x_0, y_0) \, dx \, dy \, dx_0 \, dy_0, \quad (6) \]

where \( \mathcal{K}(x_0, y_0; x, y) \) represents the transmission function of the system. In our case and assuming that the system is isoplanatic, it is given by

\[ \mathcal{K}(x_0, y_0; x, y) = K_0(x_0 - \mu x, y_0 - \mu y) \exp \left[ iK \left( \frac{x_{0d}}{z_d} + \frac{y_{0d}}{z_d} \right) \right], \quad (7) \]

where \( K_0(x_0 - \mu x; y_0 - \mu y) \) is the point spread function of the system and \( \mu \) is the magnification involved.

Substituting equation (3) in (5), and (5), (7) into equation (6) and if we assume that the field of view is sufficiently small for \( \psi \) to be approximately zero, we have

\[ J_1(x_i, y_i; x'_i, y'_i) = \frac{1}{2z_s^2} \int \int \int \int \int \text{Circ} \left( x_{0s}, y_{0s} \right) \times \exp \left\{ -iK \left[ \frac{x_{0s}}{z_s} + \frac{(y_0 - y'_0) y_{0s}}{z_s} \right] \right\} \times \exp \left\{ iK \left[ \frac{x_{0d} + (y_0 - y'_0) y_{0d}}{z_d} \right] \right\} K_0(x_0 - \mu x, y_0 - \mu y) \times \mathcal{K}(x'_i - \mu x'_i, y'_i - \mu y'_i) \, dx_0 \, dy_0 \, dx_{0d} \, dy_{0d}, \quad (8) \]

Introducing the change of variables

\[ \Delta x_0 = x_0 - x'_0, \]
\[ \Delta y_0 = y_0 - y'_0, \]
\[ (9) \]
the image intensity can be written as

\[ I_i(x_i, y_i) = \frac{1}{z_s^2} \int \int dx_0 \, d\Delta x_0 \, dy_0 \, d\Delta y_0 \int \int dx_{os} \, dy_{os} \, \text{Circ} (x_{os}, y_{os}) \times \exp \{ i \vec{K} \phi(x_0, \Delta x_0; y_0, \Delta y_0) \} G(x_0, \Delta x_0; y_0, \Delta y_0), \]  

where

\[ \phi(x_0, \Delta x_0; y_0, \Delta y_0) = -\left[ \Delta x_0 \left( \frac{x_{os} - x_{0d}}{z_s} \right) + \Delta y_0 \left( \frac{y_{os} - y_{0d}}{z_s} \right) \right] + \cos (\theta_1 \cdot \hat{n}) + \cos (\theta_2 \cdot \hat{n}) \right] \zeta(x_0, y_0) - \zeta(x_0 - \Delta x_0, y_0 - \Delta y_0) \right) \]  

\[ G(x_0, \Delta x_0; y_0, \Delta y_0) = K_0(x_0 - \mu x_i, y_0 - \mu y_i) K_0^*(x_0 - \Delta x_0 - \mu x_i, y_0 - \Delta y_0 - \mu y_i). \]  

Expression (10) can then be written as

\[ I_i(x_i, y_i) = \frac{1}{z_s^2} \int \int dx_{os} \, dy_{os} \, \text{Circ} (x_{os}, y_{os}) I_0(x_{os}, y_{os}), \]  

where \( I_0(x_{os}, y_{os}) \) is given by

\[ I_0(x_{os}, y_{os}) = \int \int \int G(x_0, \Delta x_0; y_0, \Delta y_0) \times \exp \{ i \vec{K} \phi(x_0, \Delta x_0; y_0, \Delta y_0) \} \, dx_0 \, d\Delta x_0 \, dy_0 \, d\Delta y_0. \]  

This integral can be evaluated using the method of stationary phase [4]

\[ I_0(x_{os}, y_{os}) = \sigma_0(x_v, y_v) G(x_v, \Delta x_v, y_v, \Delta y_v) \exp \{ i \vec{K} \phi(x_v, \Delta x_v; y_v, \Delta y_v) \}, \]  

when

\[ \frac{\partial \zeta}{\partial x_0} = \zeta'(x_v, y_v) = -\left( \frac{x_{os}/z_s - x_{0d}/z_s}{\cos (\theta_1 \cdot \hat{n}) + \cos (\theta_2 \cdot \hat{n})} \right) = \tan \left[ \frac{1}{2} (R_1 \cdot \hat{n} - R_2 \cdot \hat{n}) \right], \]  

\[ = \tan \left[ \theta_0/2 \right], \]  

\[ I_0(x_{os}, y_{os}) = 0, \quad \text{otherwise}, \]  

where \((x_v, y_v)\) is the critical point, where \( \partial \zeta / \partial x_0 = \partial \zeta / \partial y_0 = 0 \) and \( R_1 \cdot \hat{n} = \theta_{0s}, R_2 \cdot \hat{n} = \theta_{0d} \). However,

\[ \sigma_0(x_v, y_v) = \frac{2\pi}{k (\cos \theta_{0s} + \cos \theta_{0d}) |\zeta''(x_v, y_v)|}. \]  

If we know that \( \theta_{0s} = 2 \tan^{-1} [\zeta'(x_v, y_v)] + \theta_d \), because \( \theta_{0d} \approx \theta_d \), we can write

\[ I_0(\theta_{0s}, \phi_{0s}) = \sigma(x_v, \Delta x_v, y_v, \Delta y_v) \delta(\theta_{0s} - \{ \theta_d + 2 \tan^{-1} [\zeta_{x_0}(x_v, y_v)] \}) \times \delta(\phi_{0s} - \{ \phi_d + 2 \tan^{-1} [\zeta_{y_0}(x_v, y_v)] \}), \]  

where

\[ \sigma(x_v, \Delta x_v, y_v, \Delta y_v) = \sigma_0(x_v, y_v) G(x_v, \Delta x_v, y_v, \Delta y_v) \exp \{ i \vec{K} \phi(x_v, \Delta x_v; y_v, \Delta y_v) \}. \]
So,
\[ I(x_i, y_i) = \frac{\sigma(x_i, \Delta x_i; y_i, \Delta y_i)}{\varepsilon_0^2} \int \int \left\{ \frac{[(\theta_{os} - \theta_s)^2 + (\phi_{os} - \phi_s)^2]^{1/2}}{\beta} \right\} I_0(\theta_{os}, \phi_{os}) \, d\theta_{os} \, d\phi_{os}, \]
\[ = \frac{\sigma(x_i, \Delta x_i; y_i, \Delta y_i)}{\varepsilon_0^2} \begin{cases} 1, & \text{when } |\theta_{os} - \theta_s| \leq \beta/2 \\
0, & \text{another case} \end{cases}. \] (17)

Because
\[ v^2 = (\theta_{os} - \theta_s)^2 + (\phi_{os} - \phi_s)^2, \]
\[ \phi_{os} = [v^2 - (\theta_{os} - \theta_s)^2]^{1/2} + \phi_s, \]
so
\[ \delta\{\phi_{os} - [\theta_{os} + 2 \tan^{-1}(\zeta_{x})]\} = \delta\{\phi_s + [v^2 - (\theta_{os} - \theta_s)^2]^{1/2} - (\theta_{os} + 2 \tan^{-1}(\zeta_{x}))\}, \]
valid for $|\theta_{os} - \theta_s| \leq \beta/2$, and
\[ \delta\{\theta_{os} - [\theta_{os} + 2 \tan^{-1}(\zeta_{x})]\} = \delta\{\theta_s + [v^2 - (\phi_{os} - \phi_s)^2]^{1/2} - (\theta_{os} + 2 \tan^{-1}(\zeta_{x}))\}, \]
valid for $|\phi_{os} - \phi_s| \leq \beta/2$.

The constants of the left of the rect function become one when we clipped the data.

For instance, the term
\[ |\theta_{os} - \theta_s| \leq \frac{\beta}{2}, \] (18)
can be written as
\[ \left\{ \begin{array}{l} |\theta_{os} + 2 \tan^{-1}(\zeta_{x}) - \theta_s| \leq \frac{\beta}{2} \\
-\frac{\beta}{2} \leq \theta_{os} + 2 \tan^{-1}(\zeta_{x}) - \theta_s \leq \frac{\beta}{2} \end{array} \right\} \] (19)
if we know that $\zeta_{x}(x_s) = \tan \alpha$, where $\alpha$ represents the angle between the $x$ axis and the surface, we write
\[ -\frac{\beta}{4} + \frac{\theta_s - \theta_d}{2} \leq \alpha \leq \frac{\beta}{4} + \frac{\theta_s - \theta_d}{2}, \]
\[ \tan\left(\frac{\theta_s - \theta_d - \beta}{2}\right) \leq \tan \alpha \leq \tan\left(\frac{\theta_s - \theta_d + \beta}{2}\right), \] (20)
if $\gamma = (\theta_s - \theta_d)/2$,
\[ \tan\left(\frac{\gamma - \beta}{4}\right) \approx \tan \gamma - (1 + \tan^2 \gamma)\frac{\beta}{4}, \]
\[ \tan\left(\frac{\gamma + \beta}{4}\right) \approx \tan \gamma + (1 + \tan^2 \gamma)\frac{\beta}{4}. \] (21)
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If \( M = \tan \alpha, \ M_0 = \tan \gamma, \)

\[
M_0 - (1 + M_0^2)\frac{B}{4} \leq M \leq M_0 + (1 + M_0^2)\frac{B}{4},
\]

(22)

which are the values used in one dimension. The same analysis is made for the other direction. This analysis shows us that the glitter function in two dimensions is represented as

\[
B(M_x, M_y) = \text{Circ}(M_x, M_y),
\]

(23)

where the interval for \( M_x \) and \( M_y \) is given by the last expression.

3. Conclusions

In this work, the bidimensional source was described by a circular function, \( \text{Circ}(x_0, y_0), \) which can also be represented by a rect function in polar coordinates. Applying the van Cittert-Zernike theorem, I show that the glitter function in the bidimensional case is given by equation (23), where the radius in \( x \) or \( y \) direction is given by equation (22). This result comes from considering the source with a uniform intensity. The significance of this result is that the bidimensional glitter patterns obtained in the image can be totally described by equations (22) and (23).

As we can see, equation (22) contains information of the angular extent of the source, \( \beta, \) and of the position of the source and the detector.

In fact, if we make a projection of this bidimensional source on the detector or camera after reflection, taken into consideration the slopes of the surface and the geometrical optics, the same result is obtained. This geometrical analysis is shown in [1] for the unidimensional case.

References