Optical analysis of a simulated image of the sea surface

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A simulated sea surface with a Pierson-Neumann power spectrum was generated by a numerical model. The image was recorded on photographic film by means of a microdensitometer with a writing mode. To obtain the bidimensional power spectrum of this simulated image of the sea surface, a coherent optical system was used. This power spectrum has information about frequencies in the highest energy peak and the direction that the waves have at a specific time. The Pierson-Neumann power spectrum used to generate the simulated sea surface was compared with the bidimensional power spectrum obtained with the coherent optical system. Attenuation of the high frequencies in the measured spectrum was observed. This attenuation was probably caused by distribution of density values in the film or by the aperture of the detector used in the coherent optical system. Optical autocorrelations of the simulated sea surface were obtained, and a high degree of correlation in the direction perpendicular to the wind was found.

1. Introduction

Studies of the sea surface using optical and oceanographic methods began by the end of the last century. The radiant energy emanating from the sea surface was the first optical parameter measured. Later, measurements were made of the spectral radiance at different depths. This critical parameter together with other variables such as temperature, salinity, and nutrients determine oceanic productivity.

In the 1930s, measurements of light transmittance and scattering at different depths were made. This research allowed us to understand the particle distribution, which gives information about turbulent flows below the sea surface.

At the same time, physical oceanographers have tried to understand the structural complexities of the sea surface. They have developed techniques which permit us to obtain time series of the waves from which we can derive the power spectrum. With this information we can explain wave generation and propagation and interchanges of energy among volumes of water and also forecast wave behavior. This knowledge is used in the design and construction of harbors. The time series of wave height at a specific point on the sea surface does not give information about the wave direction. Barber used an optical method to find out the direction of the waves on the sea surface. Similar investigations have been carried out by Sugimori. He used an optical method which consists of taking photographs of the sea surface and obtaining its bidimensional optical spectrum. Since this optical method requires only photographs of the sea surface, it can be used for studying waves in deep as well as shallow waters.

The bidimensional optical spectrum contains the same information as the photographic film. There are some problems associated with the method, however, such as limitations arising from film resolution, attenuation of some frequencies, problems with the illumination and contrast in taking the aerial photographs, problems with aberrations present in the optical system, and resolution of the optical system.

The aim of this work is to analyze optically a simulated image of the sea surface (Fig. 1), studying its bidimensional spectrum, and to make a qualitative comparison with a power spectrum obtained from a real sea surface.

II. Simulation of the Sea Surface

Conceptually, the simplest way of generating random surfaces with a specific directional power spectrum is to add many sinusoids in two dimensions, each having a complex amplitude with random phase and magnitude equal to the square root of the power spectrum. Unfortunately, this process generates a small set of independent elevations of the sea surface. The reason behind this is that the method is essentially a Fourier transform of a spectrum of limited wave number con-
Fig. 1. Four simulated sea surfaces with area of 500,000 m² each. The repetition of the surfaces increases the SNR in the optical system.

The filter technique developed in linear system theory does not have this limitation.

A. Generation of the Sea Surface with the Pierson-Neumann Power Spectrum

The technique employed to produce a numerical simulation of the sea surface with a specific power spectrum is based on extension of the 1-D linear system theory to two dimensions. The main idea behind the method consists of applying a bidimensional numerical filter \( W(i,j) \) (essentially an impulse response function) to a Gaussian or white surface \( X(i,j) \) having uncorrelated random heights. \( W(i,j) \) is obtained from a numerical 2-D discrete Fourier transform of the transfer function given by the square root of the ratio of the desired output power spectrum (Pierson Neumann's in our case) to the input power spectrum corresponding to the white surface. This filter modulates the white surface producing an exit surface \( Z(i,j) \) with a specific power spectrum \( E_z(m,n) \). This power spectrum \( E_z(m,n) \) is the theoretical spectrum that has been used in the simulation of the sea surface.

We used the Pierson-Neumann spectrum (Fig. 2), because we had a great deal of information about it. This power spectrum is given by

\[
A^2(w,\theta) = \begin{cases} 
(C/w^6) \exp(-2g^2/w^2u^2) \cos^2\theta & \text{for } w_1 < w < \infty \\
0 & \text{otherwise,}
\end{cases}
\]

where \( C = 3.05 \text{ m}^2/\text{sec}^5, g = 9.81 \text{ m/sec}^2, u(\text{m/sec}) \) is the wind velocity, \( w(\text{rad/sec}) \) is the angular frequency, \( w_1 \) is the angular frequency limit which depends on the fetch and its velocity, \( \theta \) being the direction of the waves. In this case the wind direction defines \( \theta = 0 \).

Perhaps the power spectrum is the easier way to characterize a rough surface. When a surface is assumed stationary, as in the case of the sea surface, the power spectrum can be obtained from the Fourier transform of the normalized autocovariance.

The simulated sea surface presented in this work corresponds to fully developed seas in deep waters. (A fully developed sea for a fixed wind speed \( V \) is one whose spectrum contains components at all frequencies \( 0 < f < \infty \), each with the maximum energy that can be attained with the given wind speed.)

We can obtain the variance \( G \) of the surface elevations in square meters using

\[
G = 3C(\pi/2)^{2/3}u/2g^5, \tag{2}
\]

the significant wave height being \( H_{1/3} = 2.33(G)^{1/2} \) (average of the one-third highest waves in the record).

The power spectrum can be written as a function of the wave number components \( (K_x,K_y) \). The transformations are

\[
w = (gK)^{1/2}, \quad K = (K_x^2 + K_y^2)^{1/2}, \quad \theta = \tan^{-1}(K_y/K_x), \tag{3}
\]

and the Jacobian is

\[
J[(w,\theta)/(K_x,K_y)] = |(g)^{1/2}/2|K^{3/2}. \tag{4}
\]

Then the spectrum becomes

\[
E_z(K_x,K_y) = J[(w,\theta)/(K_x,K_y)]A^2[w(K_x,K_y),\theta(K_x,K_y)] \tag{5}
\]

or

\[
E_z(K_x,K_y) = (C/2g^{5/2}K^{3/2}) \exp(-2g/\sqrt{u^2K}) \cos^2\theta \tan^{-1}(K_y/K_x). \tag{6}
\]

We have only considered waves propagating in the wind direction, simplifying the equations. This stochastic model was developed initially by Caruthers and Novarini. In our work, we have amplified the model...
to include much larger areas (as much as 64 times bigger than in the initial model), and we have implemented it with a Prime 400 computer at the Centro de Investigación Científica y de Educación Superior de Ensenada (CICESE).

The numerical results of the simulation consist of a matrix that shows the elevation in centimeters of the surface relative to a mean.

B. Obtaining the Image of the Sea Surface on Photographic Film
The numerical results of the model (wave heights) were expressed as density and using a microdensitometer in the writing mode these values were written on a photographic film.

We found that Kodak Linograph Shellburst film 2476 ESTAR AH BASE gives very high contrast and has high resolution. A range of density values was written on the photographic film with the microdensitometer, and a nonlinear response was observed (Fig. 3). In view of this, the initial values were modified to compensate for the nonlinearities.

The lowest part of the wave (trough) corresponded to the minimum value of density, while the highest part (crest) corresponded to the maximum value of density, and between these extremes a linear interpolation was used to get the intermediate values. This simulation of the sea surface does not make use of the intensity of reflected sunlight as a variable, because our approximation is of the first order only. So the assignment of density values for different heights constitutes only an approximation to the real physical situation.

III. Optical Fourier Transform of the Photographic Image
To get the squared modulus of the optical Fourier transform or power spectrum of the sea images contained in the transparency, it is necessary to use a coherent optical system (Fig. 4). The optical system includes a He–Ne laser 1, a pinhole 2, and a lens \( L_1 \) of focal distance \( f \). The distance between the pinhole and the lens \( L_1 \) is equal to \( f \), thus providing collimated illumination on the object. A circular aperture 4 determines the extent of the illuminated area on the object (transparency), which is placed at 5. The object is put inside a liquid gate to increase the SNR. The lens \( L_2 \), placed at a distance \( f \) (focal length of \( L_2 \)) from the object, produces an exact Fourier transform of the object. The power spectrum obtained is measured with a photomultiplier placed at 7.

The 2-D Fourier transform at plane 7 of the complex amplitude transmittance \( t_0(x_0,y_0) \) (of the transparency) is given by

\[
U_f(x_f,y_f) = \int \int t_0(x_0,y_0) \exp(-2\pi j/x_f)(x_0 x_f + y_0 y_f) dx_0 dy_0,
\]

where \( \lambda \) is the wavelength of the light involved, \( f \) is the focal length of the transform lens, \( j = \sqrt{-1} \), and \( U_f(x_f,y_f) \) is the complex amplitude at a point \((x_f,y_f)\) in the transformation plane, \((x_0,y_0)\) being coordinates on the object plane.

IV. Optical Correlation of Two Photographic Images
A. Optical System for Correlation

The similitude of two functions can be obtained by performing a cross-correlation analysis. This measure of similitude is effective in finding out the physical structure existing in both functions.

The basic optical configuration of a coherent image-to-image correlator is shown in Fig. 5. Light coming from a laser is collimated and illuminates a section of transparency 1. The transform lens 1 produces an amplitude distribution at its back focal plane (plane 2), which is an exact Fourier transform of transparency 1. A dc block placed on the optical axis in this plane 2 permits removal of the average transmittance of

![Fig. 4. Coherent optical system for obtaining the power spectrum of the sea images contained in the transparency.](image-url)
transparency 1 allowing only the structural information to pass through. The transform lens 2 superimposes the image of transparency 1 on transparency 2 with unit inverted magnification (−1). The transform lens 3 produces the Fourier transform (plane 4) of the amplitude distribution resulting from the illumination of transparency 2 with the filtered image (that is, the image without the dc component) from transparency 1. The correlation signal appears on the optical axis, in the Fourier plane of transform lens 3, and the other light distributions are discriminated by the pinhole in front of the detector.

Correlations between two identical photographic images of the simulated sea surface were performed by illuminating a circular area on transparency 1 (12.56 mm²) and moving it on its own plane. The displacements were small with intervals Δx = 100 μm and Δy = 100 μm. In this way the values of Cff (the correlation function) were obtained.

B. Mathematical Description of the Optical Correlation System

The operation of the system can be described by the following equations. The amplitude distribution in plane 1 is

\[ a_1(x_1,y_1) = t_{01} + t_{11}(x,y), \]

where \( t_{01} \) is the average amplitude transmittance of transparency 1, and \( t_{11}(x,y) \) is the amplitude transmittance corresponding to the structure information. The amplitude distribution in plane 2 is given by

\[ a_2(x_2,y_2) = A_1(x_2/\lambda, y_2/\lambda) \]

where \( \lambda \) is the wavelength of the light used, \( f \) is the focal length of the transforming lens, \( A_1(x_2/\lambda, y_2/\lambda) \) stands for the Fourier transform of the distribution \( a_1(x_1,y_1) \).

The Fourier transform of \( t_{01} \) produces a delta function centered on the optical axis in plane 2, which is removed by the dc block. The amplitude distribution on plane 3 is

\[ a_3(x_3,y_3) = t_{11}(x_3,y_3)(t_{02} + t_{12}(x_3,y_3)). \]

Here \( t_{02} \) is the average amplitude transmittance of transparency 2, and \( t_{12}(x_3,y_3) \) is the amplitude transmittance corresponding to the image structure information. The coordinates \( (x_3,y_3) \) are defined in an inverted geometry to avoid the negative signs that occur because of the double transformation process (the image of transparency 1 being inverted).

The amplitude distribution in plane 4 is given by

\[ a_4(x_4,y_4) = A_3(x_4/\lambda, y_4/\lambda) \]

\[ a_4(x_4,y_4) = T_{02}T_{11}(0,0) + T_{11}(x_4/\lambda, y_4/\lambda) * T_21(x_4/\lambda, y_4/\lambda), \]

where \( T \) stands for the Fourier transform of \( t \), and * denotes convolution. The first term vanishes since the first dc block in plane 2 has essentially removed the information \( T_{11}(0,0) \), and hence the light passing through the pinhole to the detector is essentially given by the second term on the right-hand side of Eq. (12).

When the images corresponding to the two transparencies are identical indicating image matching, the cross correlation becomes an autocorrelation:

\[ C_{ff}(x,y) = \iint_{-\infty}^{+\infty} T_{11}(x + \Delta x, y + \Delta y) dx dy. \]

V. Results

The values of the sea surface elevation obtained from the simulation by the computer were calculated for a wind velocity of 5.0 m/sec. The surface area is ~7700 m² (Fig. 6).

The computer was programmed to produce a certain density of dots per unit area for each value of height in the waves, giving as a result a normal perspective of the simulated surface (Fig. 1).

First-order linear theory⁸ was used to calculate the wavelengths of the deep water waves with periods from 2 to 6 sec. Table I shows the wavelength in microns of the waves present in the photographic film.

Figure 7 shows the bidimensional power spectrum. The power spectrum was obtained in the optical system shown in Fig. 4. The graph of intensity against the wave number along the Xf axis is shown in Fig. 8.

The wave numbers corresponding to the points in the Fourier plane of our optical system, obtained through the relation \( X_f = \lambda/f_x \), are listed in Table II (\( f_x \) being the associated spatial frequency component).

The theoretical spectrum from the Pierson-Neumann model and the power spectrum obtained with our co-
Table I. Wavelength of the Waves

<table>
<thead>
<tr>
<th>Period (sec)</th>
<th>Wavelength (m)</th>
<th>Wavelength in the film (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>6.27</td>
<td>91.3</td>
</tr>
<tr>
<td>3.0</td>
<td>14.05</td>
<td>204.61</td>
</tr>
<tr>
<td>4.0</td>
<td>24.98</td>
<td>363.79</td>
</tr>
<tr>
<td>5.0</td>
<td>39.03</td>
<td>568.41</td>
</tr>
<tr>
<td>6.0</td>
<td>56.20</td>
<td>818.46</td>
</tr>
</tbody>
</table>

hent optical processor were normalized and plotted for comparison in Fig. 9.

Figures 10(a) and (b) show the optical autocorrelations of the sea surface along the directions parallel and perpendicular to the wind, respectively.

The correlation area was 133,000 m² corresponding to a 4-mm diam limiting aperture in front of the object.

VI. Discussion

We must make clear that the work reported in this paper has had as a main objective to analyze optically a simulated image of the sea surface (Fig. 1), studying its bidimensional spectrum, and to make a qualitative comparison with a power spectrum obtained from a real sea surface.

A calibration of the microdensitometer must be performed each time a different sea is generated on film.

Besides, the calibration must be repeated several times over a reasonable period of time to verify possible changes in microdensitometer functioning.

As mentioned before, the marine surfaces were generated using the Pierson-Neumann theoretical spec-
Table II. Localization of Sea Wave Numbers on the Fourier Plane of the Optical System

<table>
<thead>
<tr>
<th>Period (sec)</th>
<th>Wavenumber ( \frac{2\pi}{\lambda} )</th>
<th>( X_f ) Fourier plane (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0</td>
<td>1.0021</td>
<td>6.99</td>
</tr>
<tr>
<td>3.0</td>
<td>0.4472</td>
<td>3.15</td>
</tr>
<tr>
<td>4.0</td>
<td>0.2515</td>
<td>1.80</td>
</tr>
<tr>
<td>5.0</td>
<td>0.1609</td>
<td>1.17</td>
</tr>
<tr>
<td>6.0</td>
<td>0.1118</td>
<td>0.83</td>
</tr>
</tbody>
</table>

The power spectrum obtained in our optical system is of considerable value because it contains information about the wave number and about the direction of the waves with respect to the given reference system. The intensity, as a function of wave number (Fig. 8), measured in the Fourier plane by the detector is proportional to the energy assigned to the waves in the simulated marine surface, and the location or position in the Fourier plane of the frequencies contained in the spectrum corresponds to the information contained in the simulation.

Tests were made to determine the influence of exposure effects, during recording of the simulated image, on the energy spectrum obtained in the optical processor. If an adequate exposure is not achieved, it is preferable to have an underexposed slide to an overexposed one because the former gives better results for the measurements of the power spectrum by letting more energy through.

The optical system used in this work had a spatial resolution of 104 lines/mm, and it satisfactorily resolves all frequencies contained in the simulated image. Measurements of the spectrum were made only along the \( X_f \) axis, because this axis corresponds to the main direction of the waves, and most of the energy is contained here. Attenuation of the high frequencies is...
needed to generate a marine surface similar to the one
resolve the power spectrum for the high energy peak.
formation obtained.
the optical system. If we have no symmetry in the
power spectrum, we will have mistakes in the informa-
tion. The diameter of the aperture at the detector might not be uniform;
and, second, the diameter of the aperture at the detector might not be uniform;
and, finally, there might be asymmetric aberrations in
the optical system. If we have no symmetry in the
power spectrum, we will have mistakes in the information obtained.
Fortunately, our optical system can satisfactorily
resolve the power spectrum for the high energy peak.
Seventy-two hours of computing time (Prime 400) are
needed to generate a marine pattern similar to the one in
Fig. 1 with an area of 500,000 m².
One of the conditions of the theoretical model is the
nonexistence of swell which does not follow the wind
direction. This nonexistence can be observed easily in the
photograph of the power spectrum of the simulated
sea surface (Fig. 7). It can be observed that the power
spectrum lies primarily along the Xf axis. If we were
to have a sea with crossed waves, a different direction
would prominently appear in our Fourier plane.
Nevertheless, a certain widening away from the main
direction can be observed in the intensity (Fig. 7). This
can be easily explained because the locally formed waves
have an approximate fluctuation in the direction of
±30° relative to the predominant direction in some
cases.7 This is to be expected because it is included in the
model. If local waves were to follow one direction only,
the power spectrum would be confined to a very
narrow strip in the direction of the waves.

By analyzing the autocorrelation graphs (Figs. 10(a)
and (b)), a big decline is observed in the autocorrelation
values in the wind direction in comparison with the
autocorrelations values in the perpendicular direction.
This result coincides with the numerical results ob-
tained by Bruno and Novarini.4 This gives confidence in the use of the optical correlation method.
For a real sea surface (Fig. 11), uniform sun illumina-
tion does not occur. The waves are visible because they tilt the roughened sea surface alternately toward
and away from the observer thus modulating the inten-
sity of backscattered sunlight.10 However, in a real
sea surface, the slopes rather than the wave heights are
the drivers for the imaged irradiance. Figure 11 shows
an aerial picture of the sea surface taken from an alti-
tude of 300 m and from a 45° angle of inclination with
respect to the sea surface at 7:00 a.m. The altitude is,
however, a very important factor, because the higher altitudes the
better the results are when analyzing the power spec-
trum. For a higher altitude the number of waves
present is larger, and the wavelength in the film is
smaller. This means that the information in the Fou-
rier plane will appear at a certain distance away from
the central Airy pattern (dc component) thus avoiding
the noise caused by the rings in the Airy function.
Figure 12 shows the bidimensional power spectrum
of the transparency of the real marine surface. We can
see that its shape is very similar to the one optically
obtained on the basis of the theoretical model (Fig. 7).
It should be remembered that with the photographic
technique the spectra pertain to a certain very short
time interval. To obtain information of the real sea
surface using the spectrum of the photograph, it is necessary to use the method described in Denzil. The difference between both spectra is that in the last one (Figs. 12 and 13) we can observe swell in addition to sea, and swell dominates the power spectrum. In this case the wind velocity was 1.0/sec. The relevant information (Fig. 13) was obtained by measuring the energy distribution in the spectrum along the $X_f$ axis (main wind direction). Three measurements were made along the $X_f$ axis to obtain an average. The diameter of the limiting aperture in front of the object was on this occasion 3 cm.

Finally, to carry out a complete analysis, it is very important to obtain wave data by means of pressure sensors, at the same time and place where the pictures are taken, to allow for a direct comparison between this and the one obtained from the simulated image.

VII. Conclusions

In this work, optical methods for obtaining the frequency distribution contained in a marine surface have been evaluated by means of the analysis of a simulated image. The resolution of the optical system allows the analysis of all frequencies present in the simulated marine surface.

From the obtained results we may conclude that a good optical system for processing can be formed by a pair of achromatic doublets taking the proper alignment precautions.

The only limitation in carrying out this type of optical analysis concerns the size of the simulated surface because the computing time for the simulation increases in a nonlinear fashion. This affects the minimum size of the detector aperture used in scanning the optical spectrum. Also, it was found, because of the running property of the Fourier transform, that the use of several replicas of the simulated image arranged side by side in the object plane allows the use of smaller sampling apertures and increases the SNR in the system.

When comparing the power spectrum obtained optically with the Pierson-Neumann theoretical spectrum used in the simulation of the marine surface, a correlation between the two results was found to exist in the higher energy frequency range. A high correlation does not exist for the entire frequency range. It is to be noticed that the measurements have been made on one simulated marine surface only, and it is necessary to repeat the analysis on other simulated surfaces having the same wind velocity, and a different random surface to start the model, and for other seas generated with other wind velocities. The attenuation of the high frequencies in the power spectrum of the simulated sea surface can perhaps be corrected by making a detailed analysis of the relationship between wave heights and density values using this time sunlight as a parameter.

The optical correlation of the simulated marine surface in a direction perpendicular to the wind is larger than in the parallel direction. This result coincides with the numerical results obtained by Bruno and Novarini. This gives confidence in the use of the op-
tical correlation method, allowing us to extend it later to the analysis of aerial pictures of the sea surface.

In this type of analysis it will be useful to develop programs for digital processing to make an analysis similar to the one developed optically. This has the purpose of providing a more solid base for comparison between numerical and optical results.

Having aerial photographs of marine surfaces, taken under similar conditions as those corresponding to the simulated event, is indispensable for the comparison of results. This would give confidence for studying frequency distributions from real marine surfaces.

Subsequent simulations of sea surface should include proper lighting of the studied surface or a demonstration that the sunlight effects do not influence the frequency content and distribution. Besides, it would be convenient if the aerial pictures used include those regions of the sea where tests are made with pressure sensors, since that would give the real frequency distributions.

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