Experiments on barotropic vortex-wall interaction on a topographic β plane

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Abstract. The problem of a barotropic cyclonic vortex, moving on a β plane and interacting with a meridional vertical wall, is studied by means of laboratory experiments and a finite difference numerical model. In the laboratory, the vortex is produced in a rectangular rotating tank with a weakly sloping bottom. This so-called topographic β plane simulates the latitudinal variations of the Coriolis parameter (β effect). On this β plane, the cyclonic vortex moves to the northwest and eventually interacts with the western wall. Two different results are found, depending on the initial strength and zonal position of the vortex. (1) For strong vortices, opposite-sign vorticity is created at the wall owing to the no-slip boundary condition, which leads, together with the cyclone, to the formation of a dipole structure that subsequently moves away from the wall in the northeastward direction. New wall interactions may occur when the original vortex recovers its northwestward motion. (2) In the case of weak vortices, the cyclone remains approximately at the same latitude for some time and later drifts slowly southward until it is dissipated by viscous effects. It is proposed that this behavior is a consequence of the vortex dispersion due to the β effect.

1. Introduction

Considering the simple case of a single vortex near a wall, it is found that its behavior is crucially dependent on the boundary condition. For no-slip boundaries, the shear near the wall produces opposite-sign vorticity, which is able to move the vortex away from it. When considering the free-slip boundary condition, the vortex moves along the wall advected by its own image, necessary to satisfy the condition of zero normal flow. Dipole vortices colliding with a solid boundary present similar behavior. In the no-slip case, both sides of the dipole create opposite-sign vorticity near the wall and "rebound" from it in two new dipoles. For the case of a frontal vortex-wall collision, this behavior was observed by van Heijst and Flör [1989] in laboratory experiments on a stratified flow and reproduced numerically by Orlandi [1990] for two-dimensional dipoles (see also Voropayev and Afanasyev [1992] and Verzicco et al. [1995] for other type of dipole-obstacle interactions). When a free-slip wall is considered, both halves separate and move in opposite directions along the boundary, advected by their images [Saffman, 1979].

Dipole vortices possess a self-propelling mechanism, associated with the mutual interaction between both halves, enabling collisions with domain boundaries. A basic question is the following: how may a single vortex approach a wall? In geophysical applications, where rotation effects are considered, the latitudinal gradient of the Coriolis parameter (β effect) provides a self-propagation mechanism for monopolar vortices (a small dipolar component in a monopolar structure on an f plane may also cause the vortex to drift [Stern and Radko, 1998]). It is well known that barotropic cyclonic (anticyclonic) vortices on a β plane in the Northern Hemisphere move northwestward (southwestward) [see, e.g., Carnevale et al., 1991]. Because of this β-induced drift, monopolar vortices will eventually interact with the western domain boundary. A related study involving the β effect in vortex-wall collisions is that of Carnevale et al. [1997]. By means of numerical experiments, they demonstrated an inviscid mechanism due to β for the dipole rebound against an eastern boundary. When the dipole approaches the wall, the northern (southern) half displaces fluid toward the north (south), producing negative (positive) vorticity due to the conservation of potential vorticity on a β plane. Before viscous effects become important, they showed that this purely inviscid effect is able to produce the dipole rebound, analogous to the viscous rebound observed by van Heijst and Flör [1989] and Orlandi [1990].

In this paper, the interaction of cyclonic monopoles with a western wall is studied by means of laboratory experiments in a rotating tank. In order to obtain translating vortices, the β effect was mimicked by using a uniformly sloping bottom [see, e.g., van Heijst, 1994].
Because of the wall orientation and the presence of a single structure, the results are rather different from the dipole case. It is well known that vortices on a \( \beta \) plane, in addition to their propagation, radiate Rossby waves owing to the advection of fluid parcels northward (producing negative relative vorticity) and southward (producing positive relative vorticity). The result is a translating vortex with a wake of negative and positive vorticity cells. This study also examines the influence of these "modes" when the vortex is stopped by the wall.

The main results reveal two different types of behavior, depending on the vortex strength and initial position, namely, (1) strong collisions, in which the interaction of intense vortices with the wall is dominated by the production of oppositely signed vorticity at the wall due to the no-slip boundary condition, and (2) weak collisions, where the vortex dispersion due to \( \beta \) plays a significant role. An additional effect in the barotropic vortex-wall interaction, not present in the dipole case, is the mass expulsion to the left (looking offshore) for cyclonic vortices and to the right for anticyclones [Nof, 1988]. In the present study, a northward mass expulsion is produced along the wall, as has been described by other authors. This coastal flow separates from the boundary by viscous effects [Lichter et al., 1992].

In order to extend the laboratory results, a series of numerical experiments were performed by means of a finite difference code, similar to that used by Orlandi [1990], but now including rotation effects. An important advantage of the numerical tools is the study of the vortex evolution for different flow parameters, e.g., the Reynolds number and the initial separation distance of the vortex from the wall. Moreover, by performing numerical runs with a large domain, it is possible to estimate the influence of the tank size in the laboratory experiments.

The rest of the paper is organized as follows: In section 2, the experimental arrangement, the topographic \( \beta \) plane, and the vortex generation techniques are described. In section 3, the experimental results are presented; also, a scale analysis is performed in order to estimate the minimum distance that the vortex can approach the wall before being stopped by viscous effects. Section 4 shows the most important numerical results, and finally, section 5 contains a discussion and some possible applications to oceanographic cases.

2. Laboratory experiments

2.1. Rotating Tank

The laboratory experiments were performed in a rectangular rotating tank (100 cm \( \times \) 150 cm) filled with fresh tap water at a standard depth \( H_0 = 24 \) cm. The rotation rate of the tank was fixed at \( \Omega = 0.5 \) rad s\(^{-1}\), which corresponds with a Coriolis parameter \( f_0 = 2\Omega = 1 \) s\(^{-1}\). After 30 min, the spin-up is completed and the fluid has reached a state of solid-body rotation.

The creation of cyclonic vortices was performed by syphoning a finite volume of fluid through a thin, perforated tube [see Carnevale et al., 1991]. Using this method, it is possible to reproduce similar initial conditions by syphoning fixed amounts of water. On a flat bottom, the resulting vortex has vorticity and azimuthal velocity distributions that are well approximated by

\[
\begin{align*}
\omega(r) &= \frac{\Gamma}{\pi R^2} \exp\left(-r^2/R^2\right), \\
v(r) &= \frac{\Gamma}{2\pi r} \left[1 - \exp\left(-r^2/R^2\right)\right],
\end{align*}
\]

respectively. Here \( r \) is the radial distance from the center of the vortex, \( R \) is approximately the distance where the orbital velocity reaches a maximum, and \( \Gamma \) is the vortex strength (\( \Gamma/\pi R^2 \) is the maximum vorticity). Typical \( R \) values were approximately 5 cm. These vortices contain a nonzero net amount of vorticity. However, in the laboratory, it takes some seconds to create a vortex by syphoning water out of the tank; during this time, the size, strength, and position of the vortex have already been affected by the topographic \( \beta \) plane (described below). Therefore the initial experimental condition is not exactly described by (1) and (2).

The flow was visualized by adding fluorescent dye to the vortices, and the subsequent evolution was recorded with a corotating camera mounted at some distance above the tank. This method was very effective for obtaining a clear picture of the generation of vorticity next to the wall. In addition, quantitative information about the flow evolution was obtained by tracking passive tracers that were floating on the free surface. Particle tracking was carried out by using the digital image processing package DigImage [Dalziel, 1992]. With this technique, it is possible to determine the positions and velocities of a large number of tracers and interpolate them onto a rectangular grid in order to calculate the vorticity and velocity fields. Using these measurements, it is possible to visualize the large-scale radiation associated with the vortex on a \( \beta \) plane, which is more difficult to achieve when only dye is used.

2.2. Topographic \( \beta \) Plane

The evolution of a single monopole on a \( \beta \) plane may be described, in the inviscid limit, by conservation of potential vorticity for fluid columns:

\[
\frac{D}{Dt}(\omega + \beta y) = 0,
\]

where \( D/Dt \) is the material derivative and \( y \) is the meridional direction. In order to simulate the \( \beta \) effect in the laboratory, a uniform, weak, linear topography was fixed over the full length of the tank (150 cm). For this purpose, a false bottom was used, which was lifted \( \eta = 9 \) cm at one of the ends of the tank; in all the experiments, \( \tan \gamma = 9/150 \), i.e., a \( \gamma = 3.5^\circ \) slope (see
Figure 1). Conservation of potential vorticity for fluid parcels in the laboratory can be written as
\[
\frac{D}{Dt} \left( \frac{\omega + f_0}{H(y)} \right) = 0,
\]
where the bottom topography is given by \( H(y) = H_0(1 - y \tan \gamma/H_0) \). Under the restrictions of shallow topography (\( \tan \gamma \ll 1 \)) and small Rossby number flow (\( \omega/f_0 \ll 1 \)), the equation is reduced to [see, e.g., van Heijst, 1994]
\[
\frac{D}{Dt} (\omega + \beta ty) = 0,
\]
which is identical to the expression for motion on a \( \beta \) plane. The parameter \( \beta_t = f_0 \tan \gamma/H_m \) (where \( H_m \) is the mean depth of the tank) is the equivalent latitudinal gradient of the Coriolis parameter. Using \( H_m \approx 20 \) cm, the experimental value is \( \beta_t \approx 3 \times 10^{-3} \) (cm s\(^{-1}\)). These relationships show that increasing (decreasing) depth in the laboratory is equivalent to decreasing (increasing) \( y \), the latitude, in the planetary \( \beta \) plane. Then, the shallow part corresponds to "north" and the deep one to "south". Note that the restriction of small Rossby number flow is satisfied in the whole domain, except in a localized area around the experimental vortices (where the local Rossby number may be \( O(1) \), initially). However, the agreement between laboratory experiments and numerical simulations (performed on a real \( \beta \) plane, as will be shown in section 4), provides confidence in the topographic \( \beta \) plane as a valid method to mimic the planetary \( \beta \) effect [see also Carnevale et al., 1991; Masuda et al., 1989]. The \( \beta \) effect experienced by a vortex of horizontal length scale \( R \) is measured by the nondimensional number \( \beta' = \beta R/f_0 \). For the experimental vortices, this number measures typically \( \beta' \approx 0.015 \). For midlatitude, mesoscale oceanic vortices (\( f_0 \approx 10^{-4} \) s\(^{-1}\), \( R \approx 100 \) km) \( \beta' \approx 0.02 \), which is comparable to the experimental value.

The vortices gradually slow down owing to lateral diffusion of momentum and the spin-down mechanism provided by the Ekman layer at the bottom. The rate of decay induced by a sloping bottom is given by the Ekman timescale, defined as \( T_e = H_m \cos \gamma/(\nu \Omega)^{1/2} \), where \( \nu \) is the kinematic viscosity (\( \nu \approx 0.013 \) cm\(^2\) s\(^{-1}\) at 10\(^{\circ}\)C). In the experiments, the Ekman period was typically \( T_e \approx 250 \) s. On the other hand, the rate of decay by lateral diffusion is \( T_d = R^2/\nu \). This diffusion decay timescale measured typically \( T_d \approx 2000 \) s. The duration of the laboratory experiments was always less than the Ekman period \( T_e \).

3. Results

Experimentally, two different types of vortex-wall interaction behavior were observed, depending on the vortex strength: (1) strong collisions, in which the vortex behavior was dominated by the production of opposite-sign vorticity next to the wall owing to viscous effects, and (2) weak collisions, in which the dispersion due to \( \beta \) played a fundamental role in the vortex evolution. Some intermediate cases were also examined.

3.1. Strong Collisions

Figure 2 represents the typical behavior of a strong collision. The sequence of photographs shows top views of the tank, where the upper part corresponds to the north, and the vortex is visualized with bright dye (the domain \( 0 \leq x \leq 100 \) cm, \( 0 \leq y \leq 150 \) cm; it is not completely shown). In correspondence with the well-known behavior of barotropic cyclones, the initial movement is to the northwest, approaching the western wall.

In this experiment, the vortex strength was approximately \( \Gamma \approx 150 \) cm\(^2\) s\(^{-1}\) and the vortex was released at \((x_0, y_0) = (40 \) cm, \( 50 \) cm). When the collision occurs, negative vorticity is created at the southwest part of the vortex, next to the wall, owing to the no-slip boundary condition (\( t=20 \) s). This negative vorticity accumulates in a patch (consisting of undyed fluid originating from the wall). This anticyclonic vorticity patch is strong enough to form, together with the original cyclone, a dipole structure.
that starts to translate in the northeastward direction 
(t=40 s). As the dipole separates from the wall, the 
positive part quickly re-acquires its northwestward, \( \beta \)-
induced drift, leaving behind the negative part (t=60-80 
s). A new collision with the wall is observed, but it is 
own at some point to the north of the previous one. For 
even stronger initial vortices, two or even three viscous 
rebounds were observed (not shown here).

In addition, a northward current is observed to 
develop along the boundary (t=20-40 s), which is a typical 
leaking effect of vortices next to a wall. The presence of 
the boundary blocks the flow and part of the vortex fluid 
leaks northward; for an anticyclonic vortex, the leaking 
effect would be southward [see, e.g., Zavala Sansón et 
al., 1998]. At higher latitudes, this northward current 
separates from the wall by viscous effects (t=40 s).
3.2. Weak Collisions

When the cyclonic vortex is less intense or when the initial position \((x_0, y_0)\) is taken far from the wall, the dispersion due to \(\beta\) becomes fundamental during the wall interaction. The associated Rossby wave radiation is difficult to observe in the dye visualization experiments because the wake behind the dyed vortex covers most of the tank. However, by using the particle-tracking technique described above, we were able to monitor the evolution of the velocity fields and that of the stream function distribution, by following passive tracers floating on the surface.

Figure 3 shows the velocity fields and the stream function contours of a weaker vortex \((\Gamma \sim 100 \text{ cm}^2 \text{ s}^{-1})\) than the one shown in Figure 2 (again, the domain is not completely shown). The initial position was \((x_0, y_0) = (50, 60 \text{ cm})\). Once the cyclone reaches the wall \((t=60 \text{ s})\), it stays there for some time, and afterward it moves slowly to the south. The negative vorticity produced at the wall by viscous effects does not grow strong enough to form a dipolar vortex structure, as in the previous

![Figure 3](image-url)
case, and hence a rebound from the wall is not observed. We will try to show that this behavior of the vortex next to the wall is due to the effect of the \( \beta \) dispersion as the collision occurs. This radiation is clearly visible from the velocity vectors; a large clockwise circulation is produced behind the vortex \((t=10-60 \text{ s}, \text{"mode 1"})\). Later \((t=60-110 \text{ s})\), a counterclockwise circulation becomes clear \(\text{"mode 2"}\). These modes move westward behind the vortex. When the cyclone encounters the wall, the negative mode 1 continues its westward motion but now shifting to the southern part of the original vortex \((t=60-150 \text{ s})\). Afterward, the positive mode 2 drifts to the northern part of the vortex, which is advected southward \((t=120-150 \text{ s})\). Thus the vortex is trapped between the first two modes, the negative circulation at the south and the positive one at the north.

In order to give a more clear image of the large-scale circulations, Figure 4 shows the streaks from the particles floating on the surface for six different times in the same experiment. It is evident that the anticyclonic
mode 1 shifts to the south of the vortex and stays there while decreasing in size \((t=90-150 \text{ s})\). Later, mode 2 drifts to the north of the vortex \((t=150 \text{ s})\). The fact that both cells, modes 1 and 2, drift southwestward and northwestward, respectively, is in agreement with the expected behavior of anticyclonic and cyclonic circulations. Finally, the vortex is advected by mode 2 and moves slowly southward. This observation is analyzed in more detail by the numerical flow simulations (section 4).

### 3.3. Intermediate Cases

In this subsection, the transition between strong and weak collisions is studied by examining two intermediate cases. As in the weak collision case, a vortex with an initial strength of approximately \(\Gamma \approx 100 \text{ cm}^2 \text{ s}^{-1}\) is produced but now for two different initial positions, both of them closer to the wall.

Figure 5 shows the velocity fields and the stream function contours when the initial position of the cyclone is \((x_0, y_0) = (20, 60 \text{ cm})\). The vortex is now closer to the wall, and a collision stronger than the weak interaction case described above can be expected. In fact, the drift is slightly to the southwest at the beginning. This is due to the short initial distance from the wall, which induces a southward drift (see section 4). As in the strong case, a patch of negative vorticity is produced at the wall, which leads to the formation of a dipole structure that subsequently moves to the northeast. However, now, this dipole is more symmetrical; that is, the negative part is comparable with the original vortex. The motion in the northeastern direction reinforces the negative half and reduces the positive one, owing to the \(\beta\) effect [Velasco Fuentes and van Heijst, 1994]. As a result, the cyclone cannot recover its northward migration. In summary, this case is still dominated by the viscous production of vorticity at the wall; however, the \(\beta\) effect on this new patch of vorticity prevents a new interaction with the wall.

A weaker interaction is obtained when the vortex starts at \((x_0, y_0) = (40, 60 \text{ cm})\). The corresponding velocity fields and stream function contours (not shown here) are very similar to those in Figure 3. When the vortex collides, however, it is shifted slightly to the north, but afterward it moves southward (see Figure 6c). The small drift to the north is attributed to the influence of mode 1, which produces a northward flow parallel to the wall, together with a slight viscous rebound, while the southward motion is associated with mode 2, as in the weak collision.

To summarize, Figure 6 shows the trajectories of the four cases described above. The crosses represent the center of the vortex every 10 s. For early and later times, a circle (solid and dashed, respectively) indicates approximately the size of the vortex, i.e., the radius at which the maximum velocity occurs. The strong collision (Figure 6a) consists of a viscous rebound toward the northeast, until the vortex recovers its northward drift. In the next case, Figure 6b, the vortex also rebounds to the northeast, but this time it does not recover its original drift to the northwest, as explained for the intermediate case in Figure 5. Note that in these two examples, there is a short, initial motion to the southwest, which is due to the production of negative vorticity at the wall when the vortex was created. The
third case, Figure 6c, shows a short drift to the north, associated with mode 1, and afterward a southward motion along the wall due to mode 2. Finally, in the weak collision (Figure 6d), the vortex stays at a fixed latitude for some time and afterward drifts slowly to the south under the influence of mode 2. Apparently, this case is not influenced by mode 1 because the negative cell has already shifted to the south when the vortex arrives at the wall.

3.4. Production of Negative Vorticity at the Wall

In this subsection, a scale analysis is performed in order to estimate the production of negative vorticity at the wall and to get a simple criterion for predicting the minimum distance that the vortex can approach the boundary. Basically, the negative vorticity produced at the wall is estimated by considering the thickness of the boundary layer. When the magnitude of the vorticity in
this new patch is of the same order (in absolute value) as the cyclone peak vorticity, the zonal drift toward the wall is stopped [Carnevale et al., 1997].

The Reynolds number, \( Re = U d / \nu \), based on the velocity induced by the vortex \( U = \Gamma / 4 \pi d \), where \( d \) is the distance from the vortex center to the wall, measures \( Re = \Gamma / 4 \pi \nu \) (in the experiments, \( Re \sim 600 - 900 \)). The boundary layer thickness is \( \delta \sim d / Re^{1/2} \) [see, e.g., Perdier et al., 1991b]; therefore the vorticity produced at the boundary layer is approximately (in absolute value) \( \omega_a \sim U / \delta = \Gamma Re^{1/2} / 4 \pi d^2 \). Comparing with the peak vorticity of the cyclone \( \omega_c \sim \Gamma / \pi R^2 \):\

\[
\frac{\omega_a}{\omega_c} = \frac{Re^{1/2}}{4} \left( \frac{R}{d} \right)^2.
\]

This relation shows that the production of negative vorticity at the wall is very small (\( \omega_a / \omega_c \ll 1 \)) when the vortex is far from the wall (\( d >> Re^{1/4} R \)). As the vortex approaches the boundary, \( \omega_a / \omega_c \) increases and becomes \( O(1) \). At that moment, it can be assumed that

\[ t=20 \text{s} \]  
\[ t=40 \text{s} \]  
\[ t=60 \text{s} \]  
\[ t=80 \text{s} \]  
\[ t=100 \text{s} \]  
\[ t=120 \text{s} \]
Figure 6. Vortex trajectories for different types of collisions are shown as follows: (a) strong collision, where \((x_0, y_0) = (20, 60 \text{ cm}), R \sim 5 \text{ cm}, \text{ and } \Gamma \sim 150 \text{ cm}^2 \text{ s}^{-1}\), where \(R\) is the distance where velocity is maximum and \(\Gamma\) is vortex strength; (b) strong intermediate case, \((x_0, y_0) = (20, 60 \text{ cm}), R \sim 5 \text{ cm}\) and \(\Gamma \sim 100 \text{ cm}^2 \text{ s}^{-1}\); (c) weak intermediate case, \((x_0, y_0) = (40, 60 \text{ cm}), \text{ and } R \text{ and } \Gamma \text{ are the same as Figure 6b}\); and (d) weak collision, \((x_0, y_0) = (50, 60 \text{ cm}), \text{ and } R \text{ and } \Gamma \text{ are the same as Figure 6b}\). The trajectories were obtained by tracking the vortex peak vorticity. The solid (dashed) circles indicate the approximate size of the vortices at the beginning (end) of the experiments.

The vortex is stopped, because the patch of new vorticity at the wall prevents its zonal migration; however, a rebound does not necessarily occur. From the experimental results, it can be established that there are two possible situations as follows: (1) for intense vortices, the formation of a dipolar structure is observed, which is able to move away from the wall (strong collisions), or (2) a balance may occur between the boundary effects pushing the vortex offshore and the \(\beta\) drift pushing it inshore (weak collisions). In the latter case, the vortex may be advected in the meridional direction by some other mechanisms, such as the Rossby cells shown in the experimental results or the southward component induced by the boundary layer.

There is a possible practical use of relation (3). Assuming \(\omega_\alpha/\omega_c \sim 1\) when the vortex has been stopped,
the minimum distance from the wall reached by the vortex can be computed:

\[ d_{\text{min}} \sim \frac{Re^{1/4}}{2} R. \quad (4) \]

The validity of this relationship can be roughly tested for the experimental results presented in Figure 6, by estimating the parameters \( \Gamma \) and \( R \) in order to obtain \( Re \) and thus \( d_{\text{min}} \) (the vortex parameters are obtained by fitting (1) to the experimental data in a least squares sense). The results can be compared with \( d_{\text{lab}} \), the observed distance at which the vortex is stopped by the wall at time \( t_s \). The corresponding values are presented in Table 1.

The estimated error in \( d_{\text{min}} \) is at least 2 cm, whereas for \( d_{\text{lab}} \) it is 1 cm. These results show good correspondence for weak collisions and a somewhat less satisfactory one for strong interactions. However, both of them give the correct order of magnitude. The discrepancies may be due to the simplicity of the arguments presented above compared with the complexity of the processes involved. For instance, the time dependence of the production of negative vorticity at the wall has not been taken into account; this problem is related to the structure and evolution of the viscous boundary layer [see, e.g., Peridier et al., 1991a, b], and this is beyond the scope of the present paper. Besides, it is possible that the \( \beta \) effect reduces \( \omega_b \), because the flow in the boundary layer is directed southward; therefore fluid columns are stretched. In other words, \( d_{\text{min}} \) is overestimated. There may be some other effects such as the northward leaking effect, reducing the radius \( R \), and the vortex decay by viscous effects, decreasing \( Re \).

Finally, consider a typical oceanic vortex of radius \( R = 100 \) km and vorticity \( \omega \approx 10^{-5} \text{ s}^{-1} \), i.e., a strength \( \Gamma \approx \omega R^2 = \pi \times 10^5 \text{ m}^2 \text{ s}^{-1} \). Using a horizontal eddy viscosity \( \nu_H = 100 \text{ m}^2 \text{ s}^{-1} \) [Smith, 1986], one obtains \( Re \approx 250 \) and, consequently, \( d_{\text{min}} \approx 200 \) km. This result implies that the vortex would not approach the coastline at a distance less than 200 km. Apparently, the minimum distance is around twice the vortex radius, both for laboratory and oceanographic values.

### Table 1. Characteristic Parameter Values for Strong and Weak Interactions

<table>
<thead>
<tr>
<th>Case</th>
<th>( t_s ), s</th>
<th>( R ), cm</th>
<th>( d_{\text{min}} ), cm</th>
<th>( d_{\text{lab}} ), cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td>30</td>
<td>6.0</td>
<td>16.1</td>
<td>12</td>
</tr>
<tr>
<td>Strong</td>
<td>30</td>
<td>5.7</td>
<td>14.4</td>
<td>9</td>
</tr>
<tr>
<td>Weak</td>
<td>80</td>
<td>4.0</td>
<td>8.8</td>
<td>8</td>
</tr>
<tr>
<td>Weak</td>
<td>90</td>
<td>4.1</td>
<td>9.7</td>
<td>9</td>
</tr>
</tbody>
</table>

Variables are defined as follows: \( t_s \), time; \( R \), distance; \( d_{\text{min}} \), minimum distance from wall; \( d_{\text{lab}} \), observed distance to wall.

## 4. Numerical Experiments

In this section, some numerical simulations of the laboratory experiments described above are presented. The aim is to stress the relevance of the viscous effects for the strong vortex-wall collision and of the \( \beta \) dispersion for weak interactions. Moreover, a limitation of the experimental results is the size of the tank. As was shown, the cyclonic and anticyclonic circulations that arise as a result of the \( \beta \) effect, which have an influence on the vortex evolution for weak collisions, are of the same scale of the tank. Therefore we attempt to demonstrate, by performing numerical experiments with a larger domain, that the finite size of the tank does not fundamentally affect the experimental results.

We used a finite difference code, developed by P. Orlandi, R. Verzicco, and J. H. G. M. van Geffen [see, e.g., Orlandi, 1990; van Geffen, 1998], which solves the two-dimensional Navier-Stokes equation using the \( \omega - \psi \) formulation:

\[
\frac{\partial \psi}{\partial t} + J(\omega + \beta y, \psi) = \nu \nabla^2 \omega \quad (5)
\]

where \( J \) is the Jacobian operator \([J(A,B) = A_B - A_y B_x]\) and the relation between vorticity and stream function is given by the Poisson equation \( \omega = -\nabla^2 \psi \). It must be noted that the numerical experiments were performed for a \( \beta \) plane, instead of the topographic \( \beta \) plane used in the laboratory experiments. No-slip boundary conditions were imposed for all the simulations. The resolution used for all the runs was 128 \( \times \) 128 grid points and a time step of 0.1 s. Some tests performed with a higher resolution (256 \( \times \) 256 grid points) showed very small differences.

The prescribed initial conditions were similar to those in the laboratory experiments, where the initial vorticity distribution is approximated by (1). However, recall that this equation is valid for a flat bottom; thus the topographic \( \beta \) effect breaks the symmetry of such distribution. Therefore it is not possible to produce exactly the same initial condition in the simulations. In order to test the numerical model, several cases (not shown here) were simulated using the experimental values for the viscosity \( \nu \) and \( \beta \), trying to reproduce the laboratory results. The experimental Reynolds number \((Re = \Gamma/4\pi \nu)\) was approximately 612 (considering \( \Gamma \approx 100 \text{ cm}^2 \text{ s}^{-1} \)). Good agreement was found between both types of experiments, which provides confidence in the numerical simulations to study the cases of higher Reynolds numbers and different initial zonal positions. The influence of the other walls was also explored by performing simulations with large domains.

### 4.1. High Reynolds Number Simulation

In order to investigate the influence of the viscous effects on the vortex behavior, simulations with a higher Reynolds number \((Re = 6120)\) were performed for an initial condition similar to the weak collision case de-
scribed in section 3; that is, $\Gamma = 100 \text{ cm}^2 \text{ s}^{-1}$, $R = 3$ cm, and $(x_0, y_0) = (45, 50 \text{ cm})$. The radius and the initial position were taken somewhat smaller in order to let the initial symmetrical vortex adjust to the nonsymmetrical translating vortex on the $\beta$ plane. The stream function contours are shown in Figure 7 and correspond to the same times of the experimental weak collision in Figure 3. At the beginning, it is observed that the main characteristics of the experiment are qualitatively well reproduced (although, of course, the numerical vortex decays at a slower rate). The main differences are clear for later times; the collision is produced slightly more to the north ($y_{\text{collision}} \sim 98$ cm at $t=90$ s) than in the experimental case (where $y_{\text{collision}} \sim 90$ cm at $t=90$ s). This is probably a consequence of the negative mode 1, which produces a northward flow parallel to the wall (see also the trajectory of the intermediate case in Figure 6c, where the vortex moves to the north when arriving at the wall). Also, the unavoidable error in the initial condition may be partly responsible for this difference. Afterward, the southward motion of the vortex in the simulation is slightly faster than in the labora-

Figure 7. Stream function contours obtained from the numerical simulation of the weak collision using a high Reynolds number ($Re = 6120$). Solid (dashed) lines are positive (negative) contours at intervals of 1.2 cm$^2$ s$^{-1}$. 
tory experiment. Plots at $t=120$ s and $150$ s clearly suggest that this southward drift is produced by the arrival of the positive large-scale circulation cell (mode 2) at the wall. These possible mechanisms are in agreement with the fact that, for higher Reynolds numbers, the Rossby modes remain intense for longer times and therefore have a greater influence on the vortex behavior when it is stopped by the wall.

4.2. Dependence on the Initial Position

The laboratory results (strong and weak collisions) are highly dependent on the initial position from the wall. For instance, when the vortex starts too close from the boundary, the interaction may be dominated by viscous effects. However, when the initial zonal position is larger, the vortex is weaker when it arrives at the wall and the $\beta$ drift plays a more important role. In this section, a set of numerical simulations using the experimental Reynolds number and the same initial vortex as in section 4.1 but changing the initial distance from the wall is presented. Figure 8 shows the trajectories of four simulations, where the initial zonal position $x_0$ was 10, 20, 35, and 45 cm, while the meridional position $y_0$ was kept at 50 cm in all cases. The simulations are identified hereafter as s10 for $(x_0, y_0) = (10, 50)$, s20 for $(x_0, y_0) = (20, 50)$, etc.

The strong collision is clearly observed in experiments s10 and s20. For the s10 case, the initial motion is toward the south, indicating that the influence of the viscous boundary layer next to the wall overcomes the $\beta$ drift right from the start of the simulation. This motion was also observed in the laboratory experiments where the initial zonal position was too close from the wall. After separation, the vortex was able to return to the wall again, as in the strong collision case shown in Figure 6a. In a final stage, however, the dipolar structure turns to the east owing to the $\beta$ effect [see also Velasco Fuentes and van Heijst, 1994]. When $x_0$ is larger (simulation s20), the initial influence of the wall diminishes and $\beta$ induces an additional motion to the west. When viscous effects dominate again, the rebound is produced toward the northeast.

For $x_0 = 35$ cm, the initial northwestward $\beta$ drift becomes more important than the influence of the wall. Thus the initial motion is to the northwest. The interaction with the wall shows a short drift to the northwest, which seems to be a consequence of mode 1 (as explained in section 4.1, and in Figure 6c), together with a short viscous rebound. The main difference with the strong collision is that the vortex does not move southward before rebounding, and the influence of the $\beta$ radiation starts to play a role. Finally, simulation s45 represents a weak collision. Initially, the vortex moves in the northwestern direction, free from the wall influence. As the vortex gets closer to the boundary, it is advected to the south owing to Rossby mode 2, as was shown (note that in this case, mode 1 does not advect the vortex to the north; this is because it has already moved south from the vortex). It might be, however, that still a slightly viscous rebound is produced since creation of negative vorticity at the wall is always present. Simulation s45 gives results that are comparable with those presented in Figure 6d.

There is a qualitative difference between laboratory and numerical experiments for strong collisions. The numerical trajectories in simulations s10 and s20 show a cyclonic loop when the vortex rebounds, not present in the laboratory experiments. This difference is mainly due to the initial condition imposed in the numerical model. At $t=0$, the vortex is so close to the wall that the produced opposite-sign vorticity advects the vortex to the south almost immediately. There are some other possible reasons for the observed differences: (1) the numerical code does not include bottom friction; (2) the numerical $\beta$ value is strictly constant, whereas the topographic $\beta$ in the laboratory experiments depends on the local depth, which is changing linearly (although these changes are very small); (3) the accuracy of the finite difference method may be insufficient [Peridier et al., 1991a].
4.3. Numerical Runs With a Larger Domain

Additional numerical simulations of the same weak collision case (Figure 3) were performed but now using a larger domain of 200 cm × 300 cm (not shown here). The grid resolution and the time step were the same as in the previous simulations. Although the general behavior is similar, some differences are evident because the size of the Rossby modes is much larger in this numerical run. For instance, when arriving at the wall, the vortex drifts a shorter distance to the south than in the experimental case, probably because mode 2 is not strong enough to advect the vortex. However, it is concluded that, although the size of the laboratory tank may have an influence, it does not drastically affect the results described in section 3.

5. Discussion

Laboratory experiments in a rotating tank were performed to study the collision of a barotropic cyclonic vortex against a western wall on a β plane. The β effect was simulated by using a uniformly sloping bottom, which forces the vortex to translate in the northward direction and, eventually, to interact with the western wall.

The results showed two different dynamical regimes:

1. Strong collisions are those in which the production of vorticity next to the wall by viscous effects dominates the vortex evolution. In this regime, a patch of negative vorticity develops at the wall, forming, together with the original cyclone, a dipole structure that starts to move in the northeast direction. For strong enough initial vortices, the created dipole is very asymmetric, the positive half being stronger than the negative one, and, eventually, it regains its northwestward drift, hence performing a new collision with the wall.

2. Weak collisions are those in which the vortex dispersion produced by the β effect (a wake of large-scale circulation cells with alternating signs) plays a major role in the vortex behavior. The first mode (mode 1) is a large-scale negative circulation behind the cyclone as it moves toward the wall. The second mode (mode 2) is a large-scale positive vorticity cell behind mode 1. When the vortex is stopped by the boundary, mode 1 drifts toward the wall and becomes concentrated south of the vortex. Meanwhile, mode 2 arrives at the wall and becomes concentrated at the northern part of the original vortex. Thus the cyclone is trapped between both cells and is gradually advected to the south by mode 2 (in some intermediate cases between strong and weak collisions, mode 1 seems to have an influence, which is observed as a short drift of the vortex to the north before it is advected to the south by mode 2). The motion of the two modes is due to their own circulations; mode 1 is anticyclonic and thus it moves southwestward, while the cyclonic mode 2 drifts in the northwestward direction.

The first regime is present when an initially intense cyclone produces strong enough shear next to the wall to form a secondary, negative vorticity patch. This effect has been shown before to occur in different situations [see Harvey and Perry, 1971; Walker et al., 1987; Orlandi, 1990; Carnevale et al., 1997]. For the second type of interactions, the production of negative vorticity at the wall is still present but relatively weak; perhaps it plays a role in the southward advection of the vortex (in addition to the advection produced by mode 2, as described before). On the other hand, an estimation of the minimum distance that a vortex can approach the wall was obtained. This minimum distance, \( d_{\text{min}} \sim Re^{1/4} R/2 \), was estimated by arguments based on the production of negative vorticity at the wall. It was shown that for laboratory and oceanic vortices, \( d_{\text{min}} \sim 2R \).

On the other hand, highly nonlinear vortices on a β plane are able to maintain their coherence as they move, before losing energy when radiating Rossby waves. Therefore such radiation may play a minor role for strong collisions, when vortices are very intense. In contrast, for weak collisions, the vortex dispersion due to β becomes more important. This assertion was confirmed by performing numerical simulations with higher Reynolds numbers. In these experiments, Rossby mode 2 remains intense for longer times and clearly advects the vortex to the south. Also, several numerical experiments, using the same initial vortex but different initial zonal positions, showed the influence of the β radiation when the vortex started far from the wall. These results also suggest that for a complete description of the vortex-wall interaction on a β plane, one must take into account a large enough initial zonal distance from the wall. The limitation of the size of the laboratory tank was also considered. However, the results of numerical simulations using a larger domain did not differ fundamentally from the experimental results.

In geophysical applications, it is expected that a competition between viscous and inviscid mechanisms will occur [Carnevale et al., 1997]. This may be the case in the weak collisions in the present study. For instance, Smith [1986] performed numerical simulations for a two-layer system, using typical parameters of oceanic anticyclonic vortices interacting with a western continental shelf. Smith [1986, Figure 6a] shows the upper layer stream function of an anticyclone colliding with a western no-slip wall, while dispersing by the β effect. Smith’s figure shows a striking resemblance to the present experimental results of the weak interaction in Figure 3 (t=90 s), with, of course, the signs of the circulations reversed (see Figure 9). On the other hand, it is known that oceanic vortices may have associated satellite rings. For example, Vukovich and Waddell [1991] reported on Loop Current anticyclonic vortices that were accompanied by a cyclonic vortex at their northern periphery when interacting with the western continental
Figure 9. (top) Stream function contours of the weak collision shown in Figure 3, t=90 s. (bottom) Smith's [1986] Figure 6a, showing the upper layer stream function contours of an oceanic anticyclone-wall interaction. In this case, the circulations are reversed; solid (dashed) lines are negative (positive) contours.

![Stream function contours](image)

Figure 9. (top) Stream function contours of the weak collision shown in Figure 3, t=90 s. (bottom) Smith's [1986] Figure 6a, showing the upper layer stream function contours of an oceanic anticyclone-wall interaction. In this case, the circulations are reversed; solid (dashed) lines are negative (positive) contours.

shelf in the Gulf of México. In that case (as in the work by Smith [1986]), it is suggested that the opposite situation with respect to the present experiments may occur; if the translating vortex, colliding with a meridional boundary, is cyclonic (anticyclonic), Rossby mode 1 will be an anticyclonic (cyclonic) gyre, placed at the southern (northern) part of the original vortex. On the other hand, it is also suggested that southward (northward) motion of cyclones (anticyclones) encountering a western meridional wall may be related to the positive (negative) Rossby mode 2.

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