A numerical study of the Lagrangian circulation in the Gulf of California

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Abstract

The advection of particles is studied numerically in order to obtain patterns of the Lagrangian circulation in the Gulf of California during the two major seasons in the gulf. A two-dimensional vertically integrated model is used to generate the velocity fields. The forcing agents are the principal lunar tide specified at the mouth of the gulf and wind stress represented of summer and winter conditions. The flow geometry, i.e., the set of stagnation points and their associated streamlines, is computed for the mean velocity field of each season. These geometries show the well-known features of the gulf's surface circulation: A cyclonic gyre during summer (anticyclonic in winter) in the northern gulf; and a up-gulf coastal jet on the mainland side during summer (down-gulf in winter) in the southern gulf. It is shown that these mean-flow geometries govern the advection of the total velocity field for time scales much longer than the tidal period. The seasonal gyre of the northern gulf effectively trap particles for periods of up to two months. The along-shore flows of the southern gulf transport particles over hundreds of kilometers, northward in summer and southward in winter. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

The Gulf of California in the Pacific northwest of Mexico is an elongated semi-enclosed sea with an average width of about 150 km and a length of about 1100 km (see Fig. 1). Its circulation is the result of low-frequency and tidal co-oscillations with the Pacific Ocean (Heuer, 1997; Ripa, 1997), momentum and heat fluxes at the sea surface (Castro et al., 1994) and the nonlinear interaction between the flow produced by these agents and the topography (Marinone, 1997). The time variability of all these forcing agents, excepting the tides, follows the two main seasons observed in the gulf: A subtropical summer from June to September and a midlatitude winter from October to May (see, e.g., Badan-Dangon et al., 1991). Consequently, the currents in the gulf vary with the tides and the seasons, too.

Tidal currents have been measured directly at various places along the gulf; they are observed to flow primarily up- and down-gulf at the tidal frequency (see, e.g., Marinone, 1999). In contrast, the seasonal
circulation has only been sketched after numerous indirect observations, as well as some direct measurements. The picture that has emerged is the following: In 'the northern gulf'—north of Ángel de la Guada and Tiburón Islands (see Fig. 1)—the surface circulation consists of a cyclonic gyre in summer and an anticyclonic gyre in winter (e.g., Lepley et al., 1975; Lavín et al., 1997); while in 'the southern gulf'—south of the islands—it consists of along-shore flows with the same seasonal variability (e.g., Merrifield and Winant, 1989).

The Lagrangian observations in the Gulf of California are scarce and of limited spatial and temporal extent. Granados-Gallegos and Shwartzlose (1974) analyzed data of over one hundred drift bottles released and recovered in the Gulf of California between 1956 and 1972. Regarding the summer circulation, they deduced a general pattern of northward flow in the southern gulf, and suggested the existence of a cyclonic gyre in the northern gulf. Regarding the winter circulation, they deduced a southward flow in the southern gulf, but had no data about the northern gulf. Álvarez et al. (1984) released six radio buoys in Canal de Ballenas (between Ángel de la Guada Island and the peninsula of Baja California) on 17 and 24 June 1982. They tracked four buoys for less than one day and only two for three days. The trajectories, oscillating along the channel's axis, were explained by the (semidiurnal) tidal current. Emisson and Alatorre (1997) released four radio buoys in the southern gulf between Topolobampo and La Paz during the period 18–26 August 1978. The drifters, located initially across the gulf, revealed the presence of a cyclonic gyre. Finally, Lavín et al. (1997) made multiple releases of five ARGOS drifters in the northern gulf during the periods 13 September–9 October 1995 and 19 February–18 April 1996. Their observations confirmed the presence of a cyclonic gyre in summer and an anticyclonic gyre in winter.

The scarceness of Lagrangian data motivated the present study, whose main objective is to find the general pattern of the surface Lagrangian circulation in the entire Gulf of California. For this purpose we use the
velocity fields computed numerically with a barotropic model forced with the principal lunar tide ($M_2$) and uniform constant wind. This barotropic model reproduces the main features of the observed surface circulation —although the baroclinic component is at least as important as the barotropic contribution (Bray, 1988; Beier and Ripa, 1999)— because the wind stress and the baroclinic forcing are in phase in the gulf (Beier, 1997). At this stage we aim at obtaining a gross picture of how passively advected particles (viz. neutrally buoyant substances) are transported around the gulf under conditions typical of each of the major seasons of the gulf. In particular, we want to identify areas where particles will be trapped for extended periods (several weeks), as well as regions where particles will experience large excursions (hundreds of kilometers). Although we could achieve these goals by analyzing the statistics of large ensembles of numerically computed trajectories, we prefer to take the advantageous ‘geometrical’ point of view. This stems from the theory of dynamical systems, which has been fruitfully used in the study of advection in fluid flows (see, e.g., Ottino, 1990). The outline of the approach is the following: (1) Identify a steady and a time-dependent component of the flow, (2) Find the stagnation points and their associated streamlines in the steady flow (fixed points and manifolds, in the dynamical-systems theory), and (3) Investigate how these ‘geometric elements’ evolve under the influence of the time-dependent flow.

In Section 2 we describe how we obtained the velocity fields, and write them explicitly as the sum of a steady and a time-dependent part. Hereafter we refer to the steady part as ‘mean velocity field’ and to the sum as ‘total velocity field.’ Depending on the numerical experiment, the mean velocity field corresponds to (a) the $M_2$ tidal residual, (b) the $M_2$ residual plus mean flow induced by down-gulf wind (winter situation), and (c) the $M_2$ residual plus mean flow induced by up-gulf wind (summer situation). In all experiments the time-periodic component is the field of tidal currents. In Section 3 we describe how we extracted the ‘flow geometry’ from the discretized velocity fields and discuss the geometry of the steady flow. In Section 4 we show that the transport of the total velocity field is dominated by the geometry of the steady component. In Section 5 we show how pieces of the steady field’s streamlines evolve under the total velocity field. Finally, we summarize and discuss our results in Section 6.

2. Numerical methods

2.1. Circulation model

We obtain the velocity fields with a nonlinear, vertically integrated, f-plane model which has been extensively and successfully used to study tides and tidally induced currents in several places (see Crean et al., 1988, for a detailed description of the model and its application to the Juan de Fuca/Georgia Strait system, and Marinone, 1997, for an implementation in the Gulf of California).

The model includes eddy diffusivity and quadratic friction at the bottom (with realistic topography discretized on a uniform grid of square cells of about 6.48 km/side). The boundary conditions are free slip at the coasts and an Orlanski radiation condition at the open boundary.

The model was forced by two agents.

(a) The $M_2$ component of the Pacific Ocean tide arriving at the mouth of the gulf. The elevation of the sea surface at each grid point of the open boundary was calculated by linear interpolation of observed elevations at San Lucas Cape on the peninsula side and Yavares and Mazatlán on the mainland side (see Fig. 1). The difference between this linearly varying elevation and the structure of an incoming Kelvin wave is negligible because the Rossby radius of deformation at the mouth is about 2000 km, whereas the width of the gulf is only about 200 km. We also neglected the effect of other tidal components because the $M_2$ has been shown to explain the largest part of the tidal current and almost all of the tidal-residual currents (Marinone, 1997).

(b) The surface stress produced by winds blowing along the gulf with a uniform speed of $\pm 5$ ms$^{-1}$. This is a simple but representative idealization of the monsoonal regime of the gulf. Winds blow up-gulf in summer and
down-gulf in winter; with significant departures from this pattern being of short spatial or temporal extent (see, e.g., Badan-Dangon et al., 1991). Hence we used a uniform wind field for each season because the structure of the wind field variations is not well known, neither in time nor in space (Douglas, 1995); and because our numerical experiments show that moderate variations of the wind field do not significantly change the overall circulation (the directions change negligibly, while the speeds change only moderately).

The model equations are explicitly solved from rest using a time step of $\Delta t = 22.5$ s, which satisfies the Courant–Friedrichs–Lewy condition. After integrating eight tidal periods, a state of equilibrium is reached in which the velocity fields do not appreciably change from one tidal-cycle to the other; nevertheless, the integration was continued until the 60th cycle. Then a harmonic analysis of the velocities corresponding to the last 30 cycles was carried out at every mesh point $(i,j)$. Finally the transverse and longitudinal components $u$ and $v$, respectively, are least-squares fitted to

$$u_{ij} = A_{ij} + C_{ij} \cos(\Omega t - \phi^u_{ij});$$

$$v_{ij} = R_{ij} + D_{ij} \cos(\Omega t - \phi^v_{ij}).$$

In these expressions $t$ is the time, $\Omega$ is the frequency of the $M_2$ tide; $(A,B)$ is the sum of the mean wind-driven flow and the residual flow produced by nonlinear interactions of tidal currents with topography; and $(C,D)$ and $(\phi^u,\phi^v)$ are the amplitudes and phases of the time dependent (tidal) field, respectively.

2.2. Advection

The trajectory of a particle which is passively advected by the flow can be obtained by integrating the advection equations:

$$\frac{dx}{dt} = U(x,y,t);$$

$$\frac{dy}{dt} = V(x,y,t).$$

where $x$ and $y$ are the transverse and longitudinal coordinates respectively. The velocity vector $(U,V)$ at the arbitrary position $(x,y)$ must be computed by interpolating the velocities $(u,v)$ which are known only at mesh points.

The numerical computation of trajectories has therefore two main error sources: the interpolation of the velocities and the integration of Eqs. (3) and (4) themselves.

The error introduced by the interpolation depends on the mesh size and on the scheme used. As mentioned above, the mesh used to solve the model equations is fine enough, we therefore maintained that choice and tried to minimize the error with an appropriate interpolation scheme. Among the various algorithms available in the literature (see, e.g., Hockney and Eastwood, 1981), we chose the following four-point bilinear algorithm:

$$\begin{bmatrix}
U(x,y,t) \\
V(x,y,t)
\end{bmatrix} = \frac{1}{\Delta^2} \begin{bmatrix}
u_{ij} & u_{i+1,j} & u_{i,j+1} & u_{i+1,j+1} \\
v_{i,j} & v_{i+1,j} & v_{i,j+1} & v_{i+1,j+1}
\end{bmatrix} \begin{bmatrix}
(\Delta - \delta x)(\Delta - \delta y) \\
\delta x(\Delta - \delta y) \\
(\Delta - \delta x)\delta y \\
\delta x\delta y
\end{bmatrix},$$

where $\delta x = x - i\Delta$ and $\delta y = y - j\Delta$, with $\Delta$ the mesh size and $(i,j)$ the lower-left corner of the cell where $(x,y)$ lies. The simpler method of equaling $(U,V)$ to the velocity at the nearest-grid-point turned out to be too inaccurate, whereas a nine-point method gave comparable results at significantly higher computational cost.

The numerical integration of Eqs. (3) and (4) alone poses no particular difficulties: there is a large number of well-tested algorithms which are able to achieve almost any desired accuracy. The combination with the
interpolation step (Eq. (5)), however, made the testing of some algorithms necessary. We did the tests with flows for which the trajectories are known in analytical form: we computed trajectories within the grid and then we compared them with the analytical solutions. We tested the Euler scheme (E) and the second- and fourth-order Runge–Kutta schemes (RK2 and RK4, respectively). We found that, if a particle travels a distance $\Delta$ in a time $N\Delta t$, then the optimum time step is such that $N = 4, 10, 150$, for the RK4, RK2 and E schemes, respectively. This means that, for the same computational cost, the second- and fourth-order Runge–Kutta schemes deliver about the same accuracy; whereas the Euler scheme demands a much smaller time step to give comparable results. Therefore, we used the RK2 scheme in all computations discussed below.

The evolution of a passively advected contour is obtained by computing the evolution of a set of particles (nodes) which lie along the contour. As the flow evolves it may happen that some nodes approach each other owing to contour shrinking, in which case the unnecessary nodes are removed. The nodes, however, usually move apart from one another owing to stretching of fluid elements; new nodes must therefore be added between the old ones in order to guarantee an accurate description of the contour. We do this by writing the coordinates of the nodes $(x$ and $y)$ as a function of the contour length (measured from some reference node) and computing a natural cubic spline for every smooth segment of the curve where new nodes are needed. This method was tested using two-dimensional incompressible flows, because then any material closed contour must conserve the area it encloses. It was found that the area is preserved within $1\%$, even when the contour undergoes strong deformations.

3. Flow geometry

3.1. Basic theory

In two-dimensional steady velocity fields, like the mean flow, we can think of flow geometry in terms of critical points (where the velocity is zero) and the streamlines connecting some of them. Two kinds of critical points are distinguished.
points are of particular interest: (a) centers or elliptic points, around which streamlines are nested; and (b) saddles or hyperbolic points, where there are both incoming and outgoing streamlines (see Fig. 2 and the text below). These geometrical elements determine the advection properties of the flow in the entire domain: particles in the neighborhood of elliptic points remain there and rotate around them; whereas particles enter and leave the neighborhood of hyperbolic points. The reader interested in a detailed discussion of the material presented in this section should consult the monograph by Ottino (1990).

The stagnation points described above are computed as follows. First we locate cells where zeros of the mean velocity may occur (cells where neither $A_{ij}$ nor $B_{ij}$ have the same sign at the four corners). Then we make

Fig. 3. Same as Fig. 1 but now for the entire Gulf of California.
\((U,V) = (0,0)\) and solve Eq. (5) for \(\delta x\) and \(\delta y\), with \((u,v)\) replaced by \((A,B)\) since we are dealing with the mean velocity only.

The character of the fixed points is determined by the Jacobian matrix

\[
J = \begin{bmatrix}
\frac{\partial A}{\partial x} & \frac{\partial A}{\partial y} \\
\frac{\partial B}{\partial x} & \frac{\partial B}{\partial y}
\end{bmatrix},
\]

evaluated by a finite-difference approximation at these points. The eigenvalues of this matrix are the roots of the polynomial

\[
\lambda^2 - (A_x + B_y)\lambda + A_x B_y - A_y B_x = 0,
\]

where the subscript denotes a derivative (or its finite-difference approximation). Depending on the signs of the real and imaginary part of the roots, several types of fixed points exist (see, e.g., Óttino, 1990, for a complete classification). Here we confine ourselves to the following types.

1. Centres, when the two eigenvalues are imaginary. In this case the stagnation point is the centre of oval, concentric particle trajectories. (N.B. The eigenvalues of most points denoted as centres in this paper have a

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**Fig. 4.** Evolution of initially transverse material lines (frame a) after 6 weeks of advection. The velocity field is produced by (b) tides, (c) tides and down-gulf wind, and (d) tides and up-gulf wind.
nonzero real part; i.e., the trajectories are in fact inward or outward spirals. The real parts, however, are always negligible compared to the imaginary part.)

(2) Saddles, when the two eigenvalues are real and of opposite sign. In this case particle trajectories are further determined by the eigenvectors: Particles move away from the fixed point along the direction of the eigenvector associated with the positive eigenvalue (unstable direction); similarly, they move towards the fixed point along the direction of the eigenvector associated with the negative eigenvalue (stable direction). Since the flow is stationary, the outgoing streamlines are computed by integrating the motion of a particle initially located a short distance from the stagnation point along the unstable direction; the incoming streamlines are computed in a similar way, but the particle is located along the stable direction and the time integration is made backwards in time.

When the velocity field depends on time, as the total velocity field given by Eqs. (1) and (2), the instantaneous flow geometry may be computed at different times to denote the evolution of the flow. Streamlines, however, do not coincide with the particle trajectories which may become extremely complicated, even for these time-periodic flows.

3.2. The mean velocity field

Fig. 2 shows a detail (at the gulf’s mouth, near the Baja California coast) of the geometry of the mean flow generated by the $M_2$ and down-gulf wind (winter situation). The dots indicate the position of centres; whereas the cross indicates the position of saddles, with the thick and thin lines representing the outgoing and incoming

![Fig. 5. Longitudinal transport produced by tides and down-gulf wind, measured in areas relative to the total area of the gulf. The circles and stars represent up- and down-gulf transport, respectively. Frame (a) shows the net transport across the sections shown in Fig. 4a, after 2, 4, and 6 weeks; frame (b) shows the area advected more than 65 km away from the initial position; and frame (c) shows the area advected more than 130 km.](image-url)
streamlines, respectively. Some arrowheads have also been drawn along some streamlines to identify the flow direction.

The geometry of the circulation in the entire Gulf is shown in Fig. 3. The circulation pattern is divided in two regions by Ángel de la Guarda and Tiburón Islands. The northern region is dominated by a large anticyclonic gyre. To the south of the islands the circulation is dominated by two chains of cells separated by a meandering jet. The chain on the mainland side consists of anticyclonic cells whereas the chain on the peninsula side consists of cyclonic cells. A northward meandering jet runs between the chains of vortices, while a southward jet runs along the coast.

The same description, with the directions inverted, is valid for the circulation produced by the $M_2$ and up-gulf wind (summer situation). The residual flow induced by the $M_2$ alone is much smaller than the mean flow induced by the winds; especially in the southern half of the gulf. In this region the extremely weak and irregular flow results in a large number of fixed points and an intricate geometry.

4. Longitudinal transport by the total field

The particle transport along the gulf was estimated in the following manner. A set of drifter lines were defined across the gulf as follows: seven lines separated about 32.5 km on the northern part, one line between Ángel de la Guarda and Tiburón Islands, and nine lines separated about 65 km on the southern part (see Fig. 4a). These separations correspond to 5 and 10 grid points, respectively; a general picture of the longitudinal transport in the gulf is already clear at this resolution.

The evolution of the drifter lines within the field of mean plus tidal velocities was computed for a period of 6 weeks. Fig. 4b shows that the $M_2$ tide alone produces only slight undulations in the material lines, because the

![Graphs and diagrams illustrating the longitudinal transport by the total field.](image-url)

Fig. 6. Same as Fig. 5 but now the advecting field is produced by tides and up-gulf wind.
residual field is small even though tidal streams are strong (typically 0.02 and 0.2 ms\(^{-1}\), respectively). With the added effect of mean wind-driven currents, the advection of fluid is significantly stronger. Here we note again the division of the gulf in two regions: in the north the main feature is a rotation of the material lines, and in the south the alongshore jet produces up-gulf intrusions of fluid during summer, and down-gulf intrusions during winter (frames c and d).

These features have been quantified by evaluating the area of fluid that is transported along the gulf. For this purpose the following quantities were computed at each section defined by the initial position of the contours: \(A^0_N\), the area of fluid advected to the north; \(A^1_N\), the area of fluid advected more than 65 km to the north; and \(A^2_N\), the area of fluid advected more than 130 km to the north (these distances correspond to 10 and 20 grid points, respectively). Obviously \(A^0_N\) includes \(A^1_N\), and \(A^1_N\) includes \(A^2_N\). Similarly \(A^0_S\), \(A^1_S\), and \(A^2_S\) were also computed.

Fig. 5 shows the evolution of these quantities when the forcing wind is down-gulf. The symmetrical curves of \(A^0_N\) and \(A^0_S\) in the northern gulf (\(x < 300\) km in Fig. 5a) is produced by the gyre there. South of the islands \(A_S\) is always larger than \(A_N\), as a result of the intense along shore jet. When the forcing wind is inverted these pictures change accordingly (Fig. 6). Note, however, that \(A^0_N\) and \(A^0_S\) in the northern gulf (\(x < 300\) km in Fig. 6a) are not quite symmetric, and that the curves \(A_N\) continue growing towards the mouth.

5. The northern gulf

The results of the previous sections show that, for time intervals much longer than the tidal period, the advection is dominated by the mean field. This suggests that, although the time dependent part of the velocity

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![Figure 7](image-url) Fig. 7. Frame (a) shows the geometry of the mean circulation produced by tides and down-gulf wind in the northern gulf of California. Frame (b) shows four closed material contours constructed from the streamlines shown in (a); and frames (c) and (d) show the evolution of these contours after being advected during 4 and 8 weeks, respectively. Frames (e)--(h) are the same but for a velocity field produced by tides and up-gulf wind.
field is much stronger, some geometrical elements of the mean field may still be observed in the total field. This hypothesis is tested by selecting some elements of the mean field's geometry and letting them be advected by the total velocity field.

Fig. 7 shows the geometry of the mean circulation in the northern part of the gulf induced by the $M_2$ and down-gulf wind (frame a), and the $M_2$ tide and up-gulf wind (frame e). Several closed material contours were constructed by arbitrarily selecting segments of the streamlines which encircle the major circulation cells (frames b and f). These contours were then advected by the total velocity field (i.e., including both the mean and the tidal currents) during several weeks. After 4 weeks (frames c and g) there is only a slight deformation of some contours; and this tendency continues after 8 weeks (frames d and h). In other words, the large cells effectively trap fluid masses inside them at least for these period.

The persistence of these contours indicates that, in spite of the strong tidal currents, the circulation features of the mean field dominate the advection patterns in the long term—as compared to the tidal period. Moreover, the contour deformations occur along the unstable directions of the mean field's saddle points (e.g., compare frames a and d).

These results do not imply that stronger stretching and folding can not occur at smaller scales. As a matter of fact, if a small perturbation of higher time and space variability is added to the velocity fields we have, the stretching and folding becomes significantly more intense. In spite of this, the resultant contours resemble the contours shown here when viewed at the large scale, because most filaments are negligibly thin.

6. Conclusions

We studied the Lagrangian circulation in the Gulf of California by advecting particles with Eulerian velocity fields. These fields were generated with a barotropic numerical model forced with idealized wind and the observed tide.

Centre and saddle stagnation points, and the set of streamlines associated with the latter, were computed in order to characterize the mean currents. It turned out that these currents are divided by the islands in two regions with a distinctive geometry. The northern gulf is dominated by a centre around which there is cyclonic circulation in summer, and anticyclonic in winter. The southern gulf is dominated by a relatively strong coastal jet on the mainland side—up-gulf in summer, down-gulf in winter—and a weaker jet through the middle—in the opposite direction. Although with a different terminology, this characterization of the circulation was already known (e.g., Lepley et al., 1975; Lavín et al., 1997; Beier and Ripa, 1999).

The evolution of drifter lines located across the gulf as well as around the gyres in the northern gulf show that the long-term advection induced by the total velocity field is governed by the flow geometry of the mean currents. Indeed, the largest particle excursions occur along the mainland coast of the southern gulf, where the mean currents display a coastal jet; and the large gyres of the northern gulf's mean field are robust features which persist as particle-trapping areas when the advection is driven by the total velocity field.

In spite of the simplicity of the model, our results are in qualitative agreement with Lagrangian observations of drifters deployed in the northern Gulf of California (Lavín et al., 1997). These show that the centers of the summer and winter gyres are close to the position of the elliptic points computed in Section 3. Lavín et al. (1997) also observed that most drifters remained within the gyres for at least one month, while only a few of them escaped the region. This is in line with the results of Section 5, which show that the summer as well as the winter gyres in the northern gulf effectively trap passively advected particles for extended periods (up to two months). Finally, the results of Section 5 for the southern gulf—viz. southward flow during summer, northward flow during winter—agree with the descriptive observations of Granados-Gallegos and Schwartzlose (1974).

The main results presented here are robust; that is to say, they will appear also in more complex models, with higher resolutions and more realistic forcing. Notwithstanding this, our model certainly overlooks important processes. A non-exhaustive list includes the following: (a) Three-dimensional motions, such as upwelling; (b)
Stirring at scales smaller than the grid size used here, especially in shallow areas or areas with abrupt bathymetry changes; (c) Wind field variations, in particular the transition periods between seasons and intense, localized wind events. Work is currently underway to address some of these issues.

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